Corrections and clarifications to
Nonlinear Potential Theory on Metric Spaces,
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p. 6, l. −7. Replace "ε-net" by "5ε-net".

p. 21, l. 13–14. Replace "Then there exist upper gradients g_j so that" by "Then there is $\tilde{\rho} \in L^p(X)$ such that $g_j := g + \tilde{\rho}/j$, $j = 1, 2, \dots$, are upper gradients. If moreover $g < \infty$ a.e., then".

p. 21, l. 20–21. Replace the last two sentences by "Let finally $\tilde{\rho} = \rho + \infty\chi_{g' \neq g}$ and $g_j = g + \tilde{\rho}/j$. Then g_j is an upper gradient of f. If moreover $g < \infty$ a.e., then (1.6) holds."

p. 36. Add the sentence "Proposition 1.14 is from Cheeger [91]."

p. 39, l. 9, 10, −4. Replace f_{j_k} by f_{l_k}.

p. 46, l. 20. Delete "minimal".

p. 46, l. 21. Add the sentence "Moreover, $|\varphi' \circ u| g_u$ is a minimal p-weak upper gradient of $\varphi \circ u$, provided that φ is Lipschitz, $\varphi \circ u \in D^p(X)$ or $|\varphi' \circ u| g_u \in L^p(X)$."

p. 47, l. 1. After 2.14 add ", where the last inequality is only required to hold if $\varphi \circ u \in D^p(X)$".

p. 47, l. 8–9. Replace these lines by "Lemma 2.14 and (2.5) imply that $|\varphi' \circ u| g_u$ is a p-weak upper gradient of $\varphi \circ u$. Hence $|\varphi' \circ u| g_u \geq g_{\varphi \circ u}$ a.e. if $\varphi \circ u \in D^p(X)$, which in particular holds if $|\varphi' \circ u| g_u \in L^p(X)$, which in turn is true if φ is Lipschitz. To show the minimality in this case, observe that (2.5) and (2.4) yield"

p. 47, l. 17–18. Replace sentence by "Then g_u/u is a p-weak upper gradient of $v = \log u$, which is minimal if $v \in D^p(X)$ or $g_u/u \in L^p(X)$."

p. 61, l. 10. Insert "and $p > 1$" after "space".

p. 62, l. 13, 15. To avoid confusion between g_1, g_2 and g_j, replace g_j by $\tilde{g}_j$.

p. 64, l. 14–15. Replace sentence by "Lemma 2.37 for open E appeared in [45]." (When [49] was finalized the proof of Lemma 2.37 was omitted, and so the book is the original reference for Lemma 2.37 in the general form.)

p. 88 l. −3. Replace by

$$\begin{aligned}\infty = \biggl(\fint_B |u - u_B|^q \, d\mu \biggr)^{1/q} &= \lim_{j \to \infty} \biggl(\fint_B \min\{j, |u - u_B|^q\} \, d\mu \biggr)^{1/q} \\ &= \lim_{j \to \infty} \lim_{k \to \infty} \biggl(\fint_B \min\{j, |u_k - (u_k)_B|^q\} \, d\mu \biggr)^{1/q} \leq C \operatorname{diam}(B) \biggl(\fint_{\lambda B} g^p \, d\mu \biggr)^{1/p},\end{aligned}$$

Alternatively Fatou's lemma can be used.

p. 107, l. −2. Replace "complete" by "proper".

p. 131, l. 5. Replace $u \geq \frac{1}{2}$ by $u(x) \geq \frac{1}{2}$.

p. 145, l. 15. Replace $\|g_u^p\|_{L^p(X)}$ by $\|g_u\|_{L^p(X)}^p$.

p. 145, l. −8. Replace $C(r^p + 1)$ by $C(r^p + 1)\mu(2B)$.

p. 155, l. 19. Replace "Lemma 1.39" by "Corollary 1.39".

p. 159, l. −8. Insert "Assume that $\operatorname{supp} \mu$ is locally compact." before "If".

p. 169, l. 1. Replace 6.7 (xi) by 6.19 (x).

p. 182, l. −7. Replace "open" by "bounded open".

p. 246, l. −12. Replace [46] by [45].

p. 259, l. 5. Insert "Let $h_j = \max\{f_j, \psi_j\}$. As $h_j - f_j = (\psi_j - f_j)_+ \in N_0^{1,p}(\Omega)$, by Proposition 7.4, we have $h_j \in \mathcal{K}_{\psi_j, f_j}$." before "Let".

p. 259, l. 9–10. Replace by

$$\begin{aligned}\|w_j\|_{N^{1,p}(X)} &\le C\|g_{w_j}\|_{L^p(\Omega)} \le C(\|g_{u_j}\|_{L^p(\Omega)} + \|g_{f_j}\|_{L^p(\Omega)}) \\ &\le C(\|g_{h_j}\|_{L^p(\Omega)} + \|g_{f_j}\|_{L^p(\Omega)}) \le C(\|f_j\|_{N^{1,p}(\Omega)} + \|\psi_j\|_{N^{1,p}(\Omega)}) \le C\end{aligned}$$

p. 259, l. 12. Replace by

$$\|u_j\|_{N^{1,p}(\Omega)} \le \|w_j\|_{N^{1,p}(\Omega)} + \|f_j\|_{N^{1,p}(\Omega)} \le C(\|f_j\|_{N^{1,p}(\Omega)} + \|\psi_j\|_{N^{1,p}(\Omega)}) \le C.$$

p. 278, l. 18 Replace "$\sup_{\partial B'}$" by $\sup_{\partial B'} u$".
p. 345, l. 6 Replace "$1 < p \le n < q$" by "$1 < q \le n < p$".
p. 345, l. 18–19 Replace $\mathbf{R}^n$ by $\mathbf{R}^2$ twice.
p. 348, l. 9 Insert "$l_\gamma \le Ad(x,y)$ and" after "such that".
p. 371, [44] Add "on metric spaces" after "obstacle problem".
p. 371, [45] Add "on metric spaces" after "obstacle problem".
p. 372, [56] Add "in metric spaces" after "functions".