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THE TRANSITION FROM SECONDARY TO TERTIARY MATHEMATICS EDUCATION – A SWEDISH STUDY

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We report from a project that investigated the (dis)continuities in mathematical activities institutionalised in upper secondary and in undergraduate mathematics courses in Sweden. With a focus on students' awareness of emergent criteria for mathematical accomplishment and new pedagogic relations, we analyse how their awareness relates to academic success and career aspirations. Our data included a series of interviews with engineering students at two universities, complemented by a focus group discussion with lecturers, students' exam results, and teaching observations.

INTRODUCTION

Comparatively low pass rates and unequal participation in undergraduate mathematics programmes have long been seen as a consequence of problems many students face in the secondary-tertiary transition (Brandell, Hemmi & Thunberg, 2008; De Guzman et al., 1998; Gueudet, 2008; UNESCO, 1966). In their literature review, Jablonka, Ashjari and Bergsten (submitted) point to a broad spectrum of critical issues associated with the passage from learning mathematics at school to attending undergraduate mathematics courses, including curriculum misalignment; increased level of formalization and abstraction; unclear role of mathematics for different career paths; changes in teaching formats and modes of assessment; differences in expected study habits, pedagogical awareness of teachers, 'atmosphere' and students' sense of belonging.

In recent years, these issues have attracted increased interest of researchers who not only aim at explaining particular difficulties students typically have with a range of mathematical concepts, but also contribute with various forms of sociological and psychological analyses to an understanding of students' "identity in which persons see themselves developing due to the distinct social and academic demands that the new institution poses" (Hernandez-Martinez et al., 2011, p. 119). For example, in a critical appraisal of Wenger's distinction of modes of belongings in identity trajectories, Solomon (2007) describes the complexities of mathematical identities of undergraduate students from different programmes. Starting with an interest in students' motivation, Liebendörfer and Hochmuth (2015) draw on (social) psychology in exploring undergraduate students' autonomy in terms of their personal goals and values and perceived locus of control. Among other things, they observed that students' recognition of criteria for realising written solutions and an appreciation of the necessity for proof positively affected perceived autonomy. They also alert to a shift in authority relations in stating that "university expects students to work autonomously where students expect to be guided" (p. 9). Similarly, in comparing novice and experienced undergraduate students, Stadler et al. (2013) found that "beginners rely heavily on the teacher, while experienced students re-orient themselves from the teacher to other kinds of mathematical resources" (p. 2436), including their peers. This is in line with Sikko and Pepin's (2013) observation that some students appear to learn more from peer collaboration in tutorials or informal groups than from lectures.

Altogether, these examples point to the productivity of conceptualising students' learning in undergraduate university programmes as an apprenticeship into particular forms of mathematical activity, where the transition includes interpretations of and adaption to new pedagogic relations and emergent new criteria for accomplishment. This paper reports from a project that involved 60 engineering students and eight mathematics lecturers at two Swedish universities, which investigated these related questions: What are the continuities and discontinuities in mathematical activities institutionalised in upper secondary school and in undergraduate mathematics courses in these contexts? How do the students perceive similarities and differences between learning mathematics at upper secondary and university? How are their perceptions related to social dimensions of the transition, to their career aspirations and to their academic success?

THEORETICAL BACKGROUND

The project draws on a reworked version of elements of Bernstein's theory of pedagogic discourse and the anthropological theory of didactics (e.g. Chevallard, 1999). This is in line with the projects' interest in continuities and discontinuities in mathematical culture and pedagogic relations as well as distributive effects that may be implicated. Bernstein (e.g. 2000) made a distinction between *classification* as the principle that translates power relations between different categories of discourses and actors into pedagogic practices and *framing* that controls relations in social interaction. For characterizing a mathematics curriculum, strong classification means that there is minimal interaction between subject areas, while a 'project-based curriculum' exhibits weak classification. On the part of the students, classification values (strong or weak) translate into *recognition rules* for being able to distinguish the specialty of the practice or discourse. In the literature on the secondary-tertiary transition, changes in classification principles between school and undergraduate mathematics have been described, for example, as a shift from empirical to abstract, from informal ('intuitive') to formal or from inductive to deductive reasoning. We were interested in the *framing* with respect to overall changes in authority relationships in the pedagogic relation. In the study, we also aimed at describing differences in knowledge structure. The anthropological theory of didactics allows for the differentiation between mathematical practices and discourses by an explicit description of the underlying principles through a reconstruction of a body of reference knowledge realised in pedagogic practice. The types of problems and their solution methods (*praxis*) together with the related discursive tools and theoretical context (*logos*) constitute a unit of analysis (a *praxeology*). A description of the pragmatic and theoretical components of praxeologies is useful for characterizing the structure of an institutionalized field of knowledge in mathematics (e.g. calculus) or a subarea (e.g. integrals). During the interaction with the empirical data, some of the initial analytical categories underwent further amendments and transformations in the development of our *external language of description* (Bernstein, 2000).

EMPIRICAL STUDIES

The empirical material on which the project draws includes documents (e.g. examination papers and results), observations of teaching at the university, and interviews with students and some of their mathematics lecturers. 60 students, selected to represent different engineering programmes and achievement levels, volunteered to participate and were interviewed individually at three occasions during their first year of study. The interviews were structured by prompts inspired by previous empirical studies and analytical categories derived from the theoretical background. They lasted for about 30 minutes each and were audio-recorded. Towards the end of the year a focus group interview was conducted with eight university lecturers/ professors of mathematics with extensive experience in organising and teaching the first-year mathematics courses (including calculus and linear algebra) in which most of these students were enrolled (see Bergsten & Jablonka, 2015a). The audio-recorded session lasted for about 80 minutes and was structured by prompts related to the secondary-tertiary transition problem as conceptualised in the project. As the participants knew each other as colleagues and were a homogenous group regarding teaching experience, we expected the discussion could lead to a shared opinion but might also reveal some disagreements.

The type of mathematics

One aim of the study was to investigate the extent to which engineering students are aware of continuities and discontinuities in the pedagogic discourses of school mathematics and undergraduate mathematics and how they recognise and articulate these. In order to examine this, excerpts of 1-2 pages from four Swedish undergraduate mathematics textbooks were presented to them (in the second interview), representing a variety of expositions of introductory calculus in terms of strong/ weak classification and authority relations (between author-teacher and reader-student). We took each textbook as an instance of the realisation of a particular pedagogic discourse, which the students might recognise. The students were asked to rank these texts according to which they perceived as *most mathematical* and justify their ranking. For the analysis of the texts and the students' justifications we expanded our analytical tools by including the categories experiential/logical and textual/interpersonal meanings from social semiotics (Halliday & Hasan, 1989) and a reworked version of the *didactic layer* from Sierpinska (1997), who draws on Eco's idea of a *model reader*. In accordance with our theoretical analysis, we considered the strongly classified text with an *embedded* didactic layer as 'most mathematical', while the 'least mathematical' text was weakly classified with a *disembedded* didactic layer (see Jablonka et al., submitted). The same ranking of the 'most and least mathematical texts' was suggested by the eight lecturers in the focus group interview, where they had been given the same prompt as the students.

There was a clear correspondence between students' recognition of realisation principles for the text that was considered most mathematical (according the theoretical analysis as well as the ranking by the eight lecturers) and their success in the first year mathematics exams. More interestingly, the arguments for the rankings provided by the students differed in terms of focus. Academically successful students pertained to the content, level of technicality and textual coherence, for example in interview statements such as the following:

> These here now [the strongly classified texts] deal more with the mathematics itself … describe things within the mathematics

> It [one of the more weakly classified texts] more applies mathematics

> This is proof … with lots of intervals and continuous

In contrast, academically less successful students more often talked about how they experienced the texts as difficult or easy to access, and based on this provided a different ranking.

> This one I think feels clear and good …this one I like …structured and such … most academic possibly …doesn't mix so much letters and numbers but partitions it like this …so that the brain can more easily register if each stands in its own line

> Simply harder to understand … here they assume things all the time … very very much theory

Similarly, in an interview where the students were asked to mark solutions to authentic exam tasks, the high-achieving students tended to focus more clearly on the mathematical content in the solutions than the others, indicating awareness of a more developed praxeology. There were also differences in certainty regarding the level of detail required in a written solution.

In conclusion, the observed differentiation in students' interview responses to the textbook ranking prompt and exam tasks marking exercise, suggests that recognition of the classificatory principles of the undergraduate mathematics pedagogic discourse is linked to success in the examinations. The analysis also provided a nuanced description of differences in students' reflective awareness.

Benefits and career aspirations

One interview aimed at investigating the students' motives for their decision to study at university in general and to choose their engineering programme, and what value for their imagined future profession they attached to studying mathematics (see Bergsten & Jablonka, 2013). To support an analysis of potentially differential distribution, we drew on Bourdieu's (1983) notion of *cultural capital* as an *embodied* (knowledge, habitus), *objectivised* (as cultural products) or *institutionalised* (certificates, titles) form of accumulated labour. We assumed that the students' previously acquired cultural capital might account for differences in the potential gains they see from studying at university and, more specifically, mathematics. Further, we suspected that students from different engineering programmes might see their academic achievement differently, as exchangeable into economic capital, or primarily useful as institutionalised cultural capital.

The exploration showed that the students' background in the form of the family's cultural capital had indeed influenced their choice of study. In some cases, however, the cultural capital acquired during secondary education appeared to have helped to develop an orientation towards higher education. Some arguments to study at university were similar for students from different engineering programmes, such as getting a good job and that it was "a natural step":

> You can get a somewhat better job … that was the main thing

> It [to study at university] felt well like the only real alternative… didn't know what else to do after leaving upper secondary… and then yes the parents always encouraged you to go on studying

The choice of study direction reflected other types of goals, such as contributing to society by protecting the environment, becoming a boss, or an interest in the engineering activity itself:

> You do want to influence and make the world to something better in some way

> Quite a lot about leadership in this programme … you kind of want to become a boss and then it felt like a good alternative

> The interest in technology has always been there … with CAD and construction

However, more than directly useful for other subjects or in their envisaged professions, students appreciated the role of mathematics for other reasons, for example to provide "understanding":

> I don't think one will use it so much but yet I think it is good for the understanding

A general pattern that emerged in the data pointed at the students seeing their mathematics studies as providing a generic competence and/or a way of thinking:

> That is maybe not the mathematics itself but … the analy- analytical thinking and … you learn to analyse the problem in a somewhat different way than you did in upper secondary school

> … I already feel that one has kind of changed as a person by the math

The focus was not on specific mathematical skills as a form of cultural capital but rather on a 'habitus' developed by engagement with mathematics, seen as a habituation to solve problems in a rational way. In contrast to school mathematics, studying mathematics at university, according to these students, provides *mind-power* and a *meta-technology* (Bergsten & Jablonka, 2013).

Teachers' views of the transition

In the focus group interview (see Bergsten & Jablonka, 2015a), the eight experienced lecturers unanimously agreed that there exists a transition problem, even if they also agreed that there were many "good students" who did not have problems. They stated that the issue of a transition problem

> has existed all the time; one talked about this already when I started here as a doctoral student

When describing the 'problem' they mentioned decreasing pass rates and pointed to an increased "span" of individual students' knowledge in different areas; as two lecturers put it,

> A student can be very good at some things but maybe knows nothing at all about other things

> It takes longer to discover who is really good

Deficiencies in basic calculation skills and systematic thinking were seen as problematic, as well as not trying different approaches to a problem, which indicates a dependency on tutorial guidance:

> Minus signs brackets and such basic stuff can go wrong

> The probability it goes wrong at least once is pretty high if you have to make several computational steps

> Many students don't seem prepared that you may have a good idea and then we try it out and test it to see where it leads

The lecturers agreed that "lots of simple mistakes" could be observed in students' solutions to exam tasks. To them, this suggested that some students did not remember well what they had learnt in upper secondary school, as indicated in the following remark by one lecturer:

> In linear algebra one is not so much disturbed by things one does not remember from upper secondary

The "maturity" of the students was another issue of concern:

> It is also about being mature … one can't require at upper secondary that people have the same maturity maybe one needs one semester to level it out

Some aspects of lack of maturity mentioned by the lecturers included not returning assignments in time, taking teaching for granted as a kind of service, and demanding exact explanations of how many hours of study are needed to pass the course.

The group focused mostly on mathematical activities and did not address the role of mathematics for engineering careers (for example in terms of techniques for mathematical modelling); the picture that emerged was that mathematics departments aim to introduce students into strongly classified (traditional) academic mathematics. They also saw school mathematics as strongly classified but different to university mathematics in terms of knowledge structure and pedagogic relation. While in school students were seen as dependent, learning a range of techniques with the help of calculators and formularies, teachers at university expected them to create their own mathematical productions at their level of competency and without these aids. In terms of the mathematical work, this points to a more fragmented praxeology at upper secondary school with main focus on the praxis level with a less developed logos than at university. When asked about differences between secondary and tertiary mathematics the lecturers pointed at “rigour most of all”, suggesting that students face a higher level of formalisation at the transition to university mathematics. The same lecturers, however, said the level of rigour in their lectures had decreased:

> You argue for your theorems by examples that make things likely

They claimed, though, that the way in which examples were presented still supported rigour:

> The reasoning … the examples they see in lectures there the solutions are as rigorous that they no doubt would pass as solutions [on exams]

The ‘transition problems’ mentioned were framed as lying *within* the students and not related to, for example, the university mathematics curriculum or to the lecturers’ own teaching. To overcome the problems, according to these lecturers, is mainly a matter of students' own work, natural development and familiarisation. The group agreed about an increase in good ‘spirit’ in students and that most of them in the end overcome the problems pointed out:

> Enthusiasm is actually something I think has become better the last years

This group of lecturers emphasised that the quality of engineers who graduate has not deteriorated compared to the past; the difference is that students today need to "struggle more on the way".

Students’ views of the transition

In the third interview, towards the end of their first year, the students were invited to review their impressions, with a particular emphasis on the mathematics courses (see Bergsten & Jablonka, 2015b). They found studying mathematics different from other subjects regarding the bigger investment of time and effort needed, due to the special character of the subject:

> It is like night and day … the textbook in economy I read almost like fiction but the math books I rarely open … more like an encyclopedia

> High pace, more difficult to hang on, in math new things appear all the time until the exam, in other subjects you do a part and then you can let it go

> You spend a lot more time on mathematics, it feels like it is more serious

While taking a mainly individualised study approach at upper secondary school, studying together with fellow students was by most students described as an important part of their changed habits. However, throughout the year, many students tended to focus more and more on exams.

> At high school I never studied with friends and not so much at home almost only at school, now I sit a lot with friends and at home, that’s the difference

> It has changed during the year, there is more cram for the exam now than in the beginning, then you did more exercises from the textbook and you followed the textbook carefully

When comparing mathematics studies at university to upper secondary school, it was often pictured as more time consuming than expected and more demanding, both in terms of pace and effort:

> I was a little chocked the first period about how quickly it all ran … one week at high school is about two days here at most or even one day … it was very much higher pace

> The math studies are much more difficult than expected, much more to do …

Some students coming to university seemed to have been prepared for this:

> Had heard that many had problems with math, had tuned myself to that it might become tough and go for it from the start

The role of the teachers was seen as important both at school and university but in different ways.

> No demands from the teachers, they are here to help you, more whip at upper secondary school, more your own responsibility here

> Not as much contact like at upper secondary school but very nice, more on your own initiative, if you don't ask they will not explain

The special social opportunities offered at university ('student life') were generally appreciated, by some pointed at as an important part of their success:

> I like it here, one of the reasons why it has run well, you are forced to learn to know more people to study at university, is among the best there is

Overall, despite big efforts and occasional disappointments with their mathematics studies, most students described the first year as a highly valuable experience, some pointing to an increased autonomy as a critical issue and a trust that it will pay off.

> I still think it is quite good that we have all of it [the maths] during the first year even if it has been tough

> Can be nice to make your own decisions, this is quite good

> Incredibly rewarding to study at university, socially as well

> What a feeling a mountain to climb ... it's probably worth it because it will be good once you get out on the other side

CONCLUSIONS

The first year mathematics curriculum at the two institutions did not specialise towards any of the engineering programmes, but was oriented towards the internal development of mathematical theory. The students did in general not oppose to what might look like a lack of relevance, but appeared to reproduce a discourse about mathematics for developing generic problem solving skills and thinking tools that provide a logos for rationalising engineering practices. A successful transition, in our study in the eyes of both the students and the mathematicians, involved moving from a student position dependent on the pedagogic authority of the school teacher, with limited access to the principles, to a more autonomous position, based on recognition of the classificatory principles of the mathematical discourse developed in undergraduate textbooks and lectures, and needed for the successful completion of exam tasks (cf. Liebendörfer & Hochmuth, 2015). The academically less successful students appeared alienated by the level of technicality and formality and the lack of guidance. In compliance with the view that mathematics must be expected to be less accessible than most of their other university subjects during the first year, many students spent a lot of time and effort in working with the course material, often in self-initiated small group collaborations (cf. Sikko & Pepin, 2013; Stadler et al., 2013). For many, participation in social activities available for first year university students seemed to function as a backup for these efforts. Most of the students in this study described studying at university as a rewarding experience, and only a few, in their reflective accounts, focussed on the difficulties they experienced in the passage from learning mathematics at school to attending undergraduate mathematics courses. This is in line with the lecturers' claim that the students, albeit often less prepared in terms of skills and study habits, appeared to be more enthusiastic than the students in previous cohorts.

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