CME 326 Jan-March 2009

Assignment 3

(Due in class February 16)

Consider the partial differential equation

$$u_t = u_{xx} + F(x,t)\ , \quad u(x,0) = f(x), \quad 0 \le x \le 2\pi. \tag{1}$$

1. Find the periodic solution for the case

$$f(x) = \sin(\omega x)\,,$$
$$F(x,t) = (\omega^2 - \alpha)\sin(\omega x)e^{-\alpha t}\,,$$

   where ω and α are constants. (Make the ansatz $u(x,t) = a\sin(\omega x)e^{-bt}$.)

2. Prove that BDF2 (see (5.12) in course reader) is both A-stable and L-stable when applied to the test equation (5.2).

3. Discretize in space by using the 2nd and 4th order difference operators in Table 4.1, and also the 4th order Padé type operator in Table 4.5. Denote these operators by Q_1, Q_2, Q_3. Write a program that solves the problem by using BDF2 on the problem

$$u_t = Q_\nu u + F\ , \quad u(0) = f, \quad \nu = 1,2,3\,.$$

4. Choose $\omega = 5$ and $\alpha = 1$ and run the program with the number of points in space determined by Table 1.2. Measure the error at $t = 1$ in the max-norm, and comment on the agreement with the table. The time intergration method is 2nd order accurate, which means that one has to choose a time step to match the space accuracy. Let me know your choices.

Motivate your answers clearly !