CME 326 Jan-March 2009

Assignment 4

(Due in class March 2)

As **the first task**, consider the IBVP problem

$$\begin{aligned} u_t + Au_x + Bu_y &= \epsilon(Cu_{xx} + Du_{yy}) + F(x,y,t) \\ Lu(0,y,t) &= g(y,t) \\ u(x,y,0) &= f(x,y) \end{aligned} \quad (1)$$

where $x \geq 0,\ 0 \leq y \leq 2\pi,\ \epsilon > 0$. The solution is periodic in y, $u(x > \bar{x}, y, t) = 0$ and

$$A = B = \begin{bmatrix} 0 & 1 \\ 1 & 0 \end{bmatrix}, \quad C = \begin{bmatrix} 0 & 0 \\ 0 & 1 \end{bmatrix}, \quad D = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}.$$

As **the second task**, consider the coupling of the two scalar advection equations

$$\begin{aligned} u_t + au_x &= 0,\ -1 \leq x \leq 0 \\ v_t + av_x &= 0,\ 0 \leq x \leq 1 \\ u(0,t) &= v(0,t). \end{aligned} \quad (2)$$

Let $u_a = \sin(2\pi(x-t))$ be the exact periodic solution.

1. Use the Laplace transform technique *and* the energy-method in (1) to determine how many boundary conditions should be imposed at $x = 0$.
2. Determine the boundary operator L in (1) such that the problem is well posed. Give as many examples of L that you can find.
3. Introduce a mesh and write up the semi-discrete formulation of problem (2) using summation-by-parts operators and the SAT-penalty formulation for the boundary and interface conditions. Do it for different operators and meshes on the two domains.
4. Prove stability of the semi-discrete formulation for (2) using the energy-method (determine the penalty parameters). This means that both the left boundary treatment and the interface must be stable.
5. In the 1st calculation use your scheme above and $u_a(-1,t)$ as boundary data at the left boundary. In the 2nd calculation, write a scheme using periodic boundary conditions and no interface. Use $u_a(x,0)$ as the initial condition. Use the 4th order operators on page 312 in C-R. For the periodic case modify accordingly. Integrate with classical explicit R-K in time. Show by calculations what accuracy you have in space for both schemes. Next, run both schemes to $t = 100$, do mesh refinement, plot the L_2 error as a function of time and discuss the result.

Motivate your answers clearly !