CME 326 Jan-March 2009

Assignment 5

(Due in class March 9)

Consider the scalar wave propagation problem,

$$\begin{aligned} u_t + au_x + bu_y &= 0, \quad (x,y) \in \Omega, \\ Lu &= g(x,y,t), \quad (x,y) \in \delta\Omega \\ u(x,y,0) &= f(x,y), \quad (x,y) \in \Omega. \end{aligned} \tag{1}$$

The wave propagation direction $\bar{a} = (a, b)$ is constant and both a and b are postive. The domain Ω has an outward pointing normal $\bar{n}$

1. Let $\Omega = [0,1] \times [0,1]$ be the unit square. Use the energy-method on (1) to determine the boundary operator L and where to impose boundary conditions.

2. Discretize (1) using high order finite difference methods (FDM) on SBP form and use penalty terms for the boundary condition. The approximation will look like,

   $$U_t + a(P_x^{-1}Q_x \otimes I_y)U + b(I_x \otimes P_y^{-1}Q_y)U = (P_x^{-1} \otimes P_y^{-1})((E_0 \otimes \Sigma_x) + (\Sigma_y \otimes E_0))(U - G).$$

   The first element in the upper left corner of E_0 is one, the rest is zero. Use the energy method and determine Σ_x and Σ_y so that the approximation is stable. You can assume that P_x and P_y are diagonal.

3. Discretize (1) using a discontinuous Galerkin (dG) method. Show how to make it conservative and stable at the element interfaces. Show also how to implement the boundary conditions in a stable way.

4. Show how the dG method can be used to derive a finite volume method (FVM). Is the FVM cell-centered or node-centered ? Hint: solutions to FVM are constant in each volume.

5. Both the FDM and dG methods above are on a SBP form and impose boundary and interface conditions weakly. Can these schemes be combined ? *Speculate* on how that could be done.

6. Choose your favourite research issue from the material in this course, think through how that research would be done and be prepared to present that in 5 minutes to the other participants in the course on the 9th and 11th of March. You are not allowed to pick the task in item 5 above.

Motivate your answers clearly !