PRODUCTION SCHEDULING IN THE PROCESS INDUSTRY

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Abstract

The purpose of this paper is to formulate an optimization model for the production scheduling problem at continuous production sites. The production scheduling activity should produce a monthly schedule that accounts for orders and forecasts of all products. The plan should be updated every day, with feedback on the actual production the previous day. The actual daily production may be lower than the planned production due to disturbances, e.g. disruptions in the supply of a utility. The work is performed in collaboration with Perstorp, a world-leading company within several sectors of the specialty chemicals market. Together with Perstorp, a list of specifications for the production scheduling has been formulated. These are formulated mathematically in a mixed-integer linear program that is solved in receding horizon fashion. The formulation of the model aims to be general, such that it may be used for any process industrial site.

Keywords:
production scheduling, process industry, real-time optimization, receding horizon, mixed-integer linear program

1 INTRODUCTION

Process industrial sites, such as sites for chemical, food, or pulp and paper processing, are often integrated sites, as described by [1]. This means that the products of some production areas become raw materials for other areas. Changing the production rate in one area may therefore affect several other areas at the site. A production schedule for an integrated site should specify how much that should be produced and sold of each product at each time, and how the inventories at the site should be used. Since the production areas are connected, finding a good production schedule may not be trivial without the use of optimization, especially at the presence of disturbances. Several suggestions of optimization models for production scheduling have been proposed. The majority of these, e.g. the proposals by [2], [3], and [4], handle the scheduling of batch processes. The state-task network (STN) representation introduced in [2] is also used by, among others, [5] and [6] to formulate production scheduling models for continuous plants. However, these studies focus on the unit level of the equipment hierarchy of a site rather than the area/site level that is relevant for the current study. A task is typically a chemical or physical transformation process such as heating or filtration ([3]). Here, the focus is on control of the product flow of intermediate and final products at a site. Since the production is continuous, the scheduling problem concerns choosing the production rates and amounts to sell of each product rather than the scheduling of individual tasks.

Perstorp is a world-leading company within several sectors of the specialty chemical market. The company has 10 production sites around the world, where each production site is divided into about 5-10 production areas. The production sites typically run in a continuous mode, without any product changes or grade changes. The aim of Perstorp is to run its production sites in a well-defined way even when there are site-wide disturbances such as disruptions in a utility or raw material. In order to do so, decision makers at Perstorp have generated a specification list containing demands and desires for the production scheduling system. This list is used as a starting point for finding a production scheduling model that is generic enough to be applied to all its production sites.

In [7], a hierarchical structure for production scheduling and disturbance management is suggested, which is briefly described in Section 2. The current paper handles the upper level of the scheduling hierarchy, the production scheduling activity. The only aspect of the lower level detailed production scheduling activity that is considered here is the information exchange with the production scheduling. The production scheduling is designed on the basis of the specification list from Perstorp. The demands and specifications are formulated mathematically in a mixed-integer linear programming model. To get decision support on how to handle disturbances in real-time, the production scheduling is run in receding horizon, where the monthly production plan is updated each day. An example of a receding horizon solution of a production scheduling problem is included in Section 5 at the end of this paper to show how production disturbances are handled by the scheduling.

2 SCHEDULING HIERARCHY

In [7], the structure depicted in Figure 1 is suggested for scheduling and disturbance management. The production scheduling layer in the figure produces a production schedule based on information on orders and forecasted orders, and the detailed production scheduling takes care of disturbances in production. Similar hierarchical scheduling strategies are suggested in [8] and [9]. This paper focuses on the production scheduling activity, and aims at describing the functions of this activity in detail as well as how the functions can be formulated mathematically. The activities that the production scheduling should handle have been formulated together with Perstorp. The resulting list of specifications is given in Section 3. The specifications are then incorporated in an optimization model in Section 4.
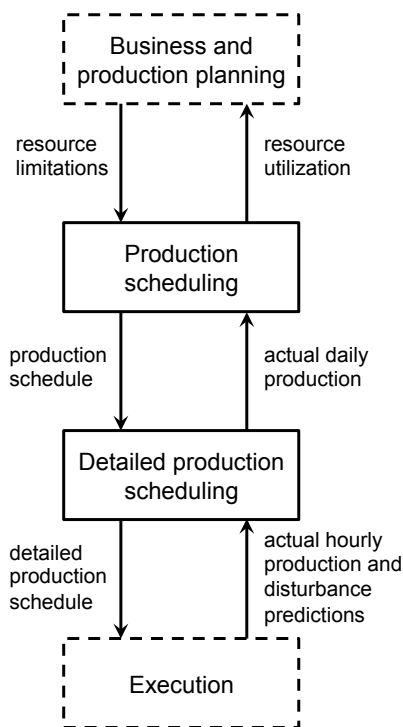
Figure 1: Suggested scheduling hierarchy.

3 PRODUCTION SCHEDULING ACTIVITIES

The objective of the production scheduling (PS) is to produce a production schedule that serves as an input to the lower level of the hierarchy in Figure 1, the detailed production scheduling (DPS). The production schedule defines reference values for the inventory levels, production rates and sales of all products for the DPS. The suggestion in [7] is to make the plan for one month, and update the plan every day in receding horizon. The current PS solution is sent to the DPS at the beginning of each day. Production scheduling is particularly interesting for integrated sites, where production areas are connected such that the products of some areas are raw materials to other areas. The product flow at integrated sites may be modeled as described in [10], as a network where directed arcs indicate possible transfers of intermediate products. An example of an integrated site is given in Figure 2. Together with decision makers at Perstorp, the following list of activities that should be handled by the production scheduling was produced.
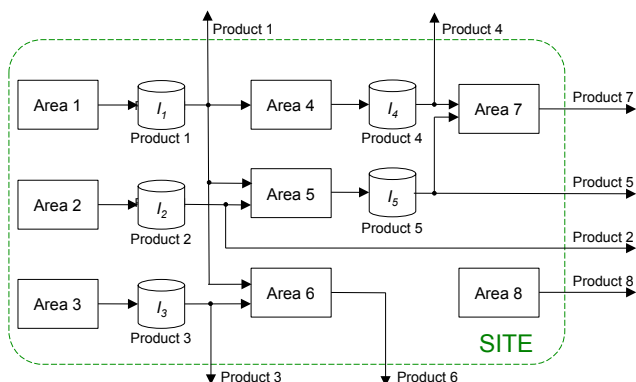

Figure 2: Example of an integrated site.

3.1 The connection of production areas

At integrated sites, as the one shown in Figure 2, production areas are connected via the flow of products at the site. The scheduling needs to take this into account when producing the production schedule.

3.2 Orders and forecasts

Information on *which* order quantities that should be produced, and *when*, should be provided as an input for the scheduling. The scheduling should be able to handle both actual orders and forecasts. The actual orders are the most important to fulfill, which should be accounted for in the scheduling. For the first days of the scheduling period, it is reasonable to assume that most of the orders are final, whereas for the last days, fewer orders have probably been placed and forecasted orders dominate. The scheduling should be performed such that the backlog of orders is as small as possible.

3.3 Production rate limitations

Production areas have a maximum capacity, which should be incorporated into the scheduling. Most areas also have a minimum rate at which they can operate, unless they are completely shut down. Production between zero and the minimum rate is thus not possible. The minimum production rate limits exist due to physical limitations, e.g. a distillation column has a minimum rate at which it can operate.

3.4 Inventory limitations

Most inventories have a maximum capacity. At some sites, this is not a strong limitation, since extra inventory capacity may be achieved by e.g. renting storage space temporary warehouses. However, at chemical plants the inventories are often (liquid) buffer tanks, which means that the maximum and minimum limits are hard constraints.

3.5 Reference intervals for inventories

In some cases, it could be desirable to keep the inventory levels at the site at certain reference levels. However, in many cases it might be a better approach to consider a reference interval, where it does not matter what the inventory level is, as long as it is kept between some minimum and maximum levels. These limits have to be within the hard constraints on the maximum and minimum inventory levels discussed previously.

3.6 Start-up costs and start-up times

Shutting down and starting up areas is often very expensive and time-consuming, which should be taken into account when performing the scheduling. The start-up costs originate from the cost of utilities and raw materials that are consumed during the start-up phase, e.g. for heating up reactors. The start-up time is the time it takes for the area to return to normal production rate after a shutdown. This time could be several days for some areas.

3.7 Market conditions

The scheduling has to consider the profitability of the forecasted orders, in addition to the confirmed orders. The scheduling should aim at maximizing the total profit.

3.8 Costs for late delivery

Late delivery of orders may lead to penalty costs. The aim of the scheduling should be to minimize the backlog of orders to avoid these costs.

3.9 Cost of production rate changes

Changing the production rate of an area quickly may be hard for the operators, and a stable production rate is also often more economically profitable because of lower average utility and raw material consumption. The scheduling should take this into account, and penalize large production rate changes from one day to the next.

4 FORMULATION OF THE OPTIMIZATION PROBLEM

The issues stated in Section 3 that should be incorporated into the optimization problem are handled one by one in this section. A suggestion on how to formulate each constraint or specification mathematically is given. The model is formulated for integrated sites, and it is assumed that each area produces one product, which is stored in one buffer tank. The set of production areas at the site is denoted A, and the set of time periods that is considered is denoted T.

4.1 Formulation of constraints

The connection of production areas

If the area dynamics are fast compared to the dynamics of the production network, the production in an area can be assumed to be directly proportional to the inflows to the area (i.e., the dynamics within the area are ignored). This can be expressed as

$$q_{ijt}^{in} = q_{jt}a_{ij} \qquad (1)$$

where q_{ijt}^{in} is the inflow of product i to area j at time t, q_{jt} the production in area j at time t, and a_{ij} is called the conversion factor between product i and product j. The connection of production areas may be expressed as mass balances at the buffer tanks at the site, i.e.,

$$I_{it} = I_{i,t-1} + q_{it} - q_{it}^{O} - q_{it}^{F} - \sum_{j\in\Omega_i} q_{jt}a_{ij}, \qquad i\in A, t\in T \qquad (2)$$

where I_{it} is the inventory level of tank i at the end of time period t, q_{it}^{O} the orders sent to the market from the buffer tank of product i at time t, q_{it}^{F} the forecasted orders sent to the market from the buffer tank of product i at time t, and Ω_i the set of areas directly downstream of area i. The notation is shown in Figure 3, which shows a site with three production areas.
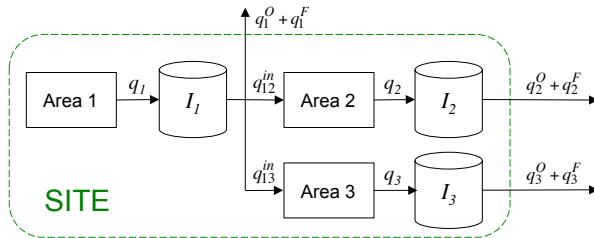

Figure 3: Example of a site with three areas.

Orders and forecasts

The order quantity of product i that should be delivered at time t is denoted O_{it} and is given as an input to the optimization, for all products at the site and all times over the horizon. A forecast of the anticipated orders over the horizon is also assumed to be given. The forecasted order quantity of product i at time t is denoted F_{it}.

Production rate limitations

To avoid solutions where the areas operate at rates between zero and the minimum rate, binary variables w_{it} may be used. w_{it} is equal to one if area i operates at time t, and zero otherwise. The production rate constraints are expressed as

$$w_{it}q_i^{\min} \le q_{it} \le w_{it}q_i^{\max}, \qquad i\in A, t\in T \qquad (3)$$

where $q_i^{\min}$ is the minimum rate at which area i can operate, and $q_i^{\max}$ the maximum production rate during one time step for area i.

Inventory limitations

The hard constraints on the inventory levels may simply be expressed as

$$I_i^{\min} \le I_{it} \le I_i^{\max}, \qquad i\in A, t\in T \qquad (4)$$

where $I_i^{\min}$ and $I_i^{\max}$ are the minimum and maximum allowed inventory levels in tank i.

Reference intervals for inventories

Auxiliary variables z_{it} may be used to put a linear penalty on deviating from the reference interval in a buffer tank. For a reference intervals with lower bound I_i^{lb} and upper bound I_i^{ub}, constraints

$$I_i^{lb} - z_{it} \le I_{it} \le I_i^{ub} + z_{it}, \qquad i\in A, t\in T \qquad (5)$$

may be formulated, and the auxiliary variables penalized in the objective function, to achieve this.

Start-up costs and start-up times

A binary variable s_{it} can be introduced to keep track of when an area has been shut down. Inequality constraints

$$s_{it} \ge w_{i,t-1} - w_{it}, \qquad i\in A, t\in T \qquad (6)$$

force s_{it} to be equal to one if area i has been shut down from time $t-1$ to t. To penalize shutdown/start-up of areas, s_{it} may be penalized in the objective function.

The start-up times for areas may be formulated as the constraint

$$w_{i,t+j} \le 1 - s_{it}, \qquad j = 1,\ldots,n_i, \qquad i\in A, t\in T \qquad (7)$$

where n_i is the start-up time of area i.

Market conditions

The forecasted orders that should be prioritized are the orders that give the highest profit. This may be achieved by penalizing $-\sum_t m_i q_{it}^{F}$ in the objective function, where m_i is the contribution margin of product i. Another constraint imposed by the market is that the flows to the market may not be greater than the demand. For the flow to the market of orders, both the orders at the time step and the backlog of orders from the previous time step have to be taken into account. The constraint may be expressed as

$$q_{it}^{O} \le O_{it} + B_{i,t-1}, \qquad i\in A, t\in T \qquad (8)$$

where $B_{i,t-1}$ is the backlog of product i at time $t-1$, which is defined under 'Costs for late delivery'.

For forecasts, the constraint

$$q_{it}^{F} \le F_{it}, \qquad i\in A, t\in T \qquad (9)$$

is used to ensure that the production scheduling does not plan to produce more of a product than can be sold.

Costs for late delivery

The backlog of orders, B_{it}, of product i at time t is given by

$$B_{it} = B_{i,t-1} + O_{it} - q_{it}^{O}, \qquad i\in A, t\in T \qquad (10)$$

where the variables $B_{it} \ge 0$ at all times. To avoid late delivery of orders, the backlog may be penalized in the objective function. To ensure that the backlog of the most profitable products is handled first, the term may be weighted by factors proportional to the contribution margins, m_i.

Cost of production rate changes

Auxiliary variables x_{it} may be used to penalize production rate changes. Constraints

$$q_{it} - x_{it} \leq q_{i,t-1} \leq q_{it} + x_{it}, \qquad i \in A, t \in T \tag{11}$$

may be formulated, and x_{it} penalized in the objective function.

4.2 Variables and parameters

The variables that were used to formulate the constraints in Section 4.1 are summarized in Table 1. All variables are continuous except for the binary variables w_{it} and s_{it}.

Table 1: Variables

I_{it}	Inventory level of tank i at the end of time period t
q_{it}	Daily production rate of area i at time t
q_{it}^O	Daily flow to the market of orders of product i at time t
q_{it}^F	Daily flow to the market of forecasted orders of product i at time t
B_{it}	Backlog of product i at the end of time period t
w_{it}	Operational mode of area i (on/off) at time t
s_{it}	Shutdown/start-up variable for area i at time t
z_{it}	Auxiliary variable for buffer tank reference interval for tank i at time t
x_{it}	Auxiliary variable for production rate changes for area i at time t

The external inputs to the model are the orders, O_{it}, and forecasted orders, F_{it}, for all products i at each time step t over the horizon. The model also needs initial conditions for the production rates, sales, and inventories. These are given to the PS as measurements from the site (information from DPS), such that the initial conditions represent the actual production and inventory levels at the start of the optimization. To handle the start-up time constraints in the receding horizon formulation, the previous n_i measurements of w_{it} are needed. This is handled by supplying the last n_i measurements of q_{it} to the PS as initial conditions. From these measurements, the initial conditions of w_{it} that are required by the PS are also given.

4.3 Formulation of objective function

As described previously, some variables have to be penalized in the objective function to handle the specifications from Section 3. The suggested objective function is

$$Z_{PS} = \sum_{i \in A} \sum_{t \in T} (\alpha_i m_i B_{it} - \beta_i m_i q_{it}^F + \gamma_i s_{it} + \delta_i z_{it} + \varepsilon_i x_{it}) \tag{12}$$

where α, β, γ, δ, and ε are penalty parameters. The first term penalizes backlog of orders, where the weighting with the contribution margins, m_i, ensures that backlog of the most profitable products is handled first. The second term encourages to plan the production for forecasted orders. The most profitable forecasted orders are prioritized, since the term is weighted with the contribution margins. The third term penalizes start-up/shutdown of areas. The weight on this variable, γ, should correspond to the cost of starting up the different production areas. The last two terms penalize the auxiliary variables z_{it} and x_{it}, to account for the buffer tank reference interval and the cost of production rate changes, respectively.

4.4 Resulting optimization problem

An optimization problem that takes the specifications in Section 3 into account may be formulated as

minimize (12)

subject to (2) – (11)

and $I_{it}, q_{it}, q_{it}^O, q_{it}^F, B_{it}, z_{it}, x_{it} \geq 0, \quad i \in A, t \in T$

$\qquad w_{it}, s_{it} \in \{0,1\}, \quad i \in A, t \in T$, (13)

The problem is a mixed-integer linear program (MILP) since it consists of both binary and continuous variables and the objective function is linear. In total, there are $7|A||T|$ continuous variables and $2|A||T|$ binary variables.

5 AN EXAMPLE

The example provided in this section is constructed to resemble a real industrial site. Real data from Perstorp cannot be published due to secrecy matters. The planning horizon in the example is a month ($|T| = 30$ days).

5.1 Background data

The site structure is depicted in Figure 4. The site has six production areas, six products and six buffer tanks. The downstream areas are most profitable, and product 3 is a byproduct for which there is no demand. The contribution margins, m_i, maximum and minimum production rates, and $q_i^{\max}$ and $q_i^{\min}$, and start-up times, n_i, for all production areas/products are given in Table 2. The buffer tanks are limited between zero and one (0-100 %) and a reference interval between 40 % and 60 % of the buffer tank volume is desired for all buffer tanks.
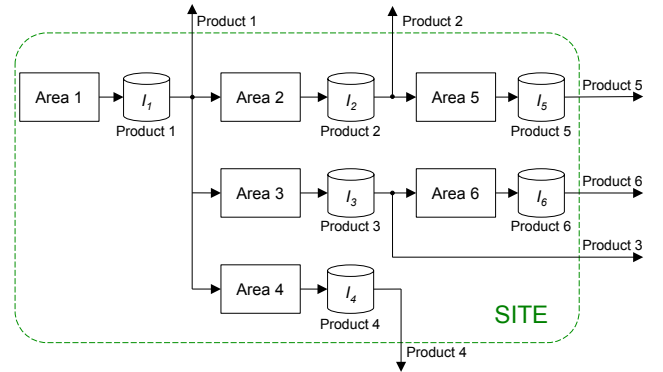

Figure 4: Site considered in the example.

Table 2: Production data

	$q_i^{\min}$, (kg/day)	$q_i^{\max}$, (kg/day)	m_i ($/kg)	n_i (days)
Product 1	2.4	24	0.4	4
Product 2	1.2	12	0.7	2
Product 3	0.48	4.8	0.1	3
Product 4	0.24	2.4	0.5	1
Product 5	0.48	4.8	0.8	2
Product 6	0.48	4.8	1	1

To make the results easier to interpret, it is assumed that the forecasts are perfect, such that all forecasted orders become actual orders. The information on all orders of the month is given already at the first day of the month. This

means that the flows to the market q_{it}^O and q_{it}^F can be merged into one flow, q_{it}^M. For simplicity, it is also assumed that all the conversion factors are equal to one, $a_{ij} = 1$. The mass balance constraints according to (2) become

$$I_{1t} = I_{1,t-1} + q_{1t} - q_{1t}^M - q_{2t} - q_{3t} - q_{4t}, \qquad t \in T \qquad (14)$$

$$I_{2t} = I_{2,t-1} + q_{2t} - q_{2t}^M - q_{5t}, \qquad t \in T \qquad (15)$$

$$I_{3t} = I_{3,t-1} + q_{3t} - q_{3t}^M - q_{6t}, \qquad t \in T \qquad (16)$$

$$I_{it} = I_{i,t-1} + q_{it} - q_{it}^M, \qquad i = 4,5,6, \ t \in T \qquad (17)$$

5.2 Shaping the objective function

The objective function that is used is given by (12). At the site, the costs for shutting down/starting up areas are very high. Thus, a large penalty on start-ups is used in the objective function. The cost of starting up/shutting down is approximately equal for all areas. It is more important to deliver orders on time (avoid backlog) than to avoid to deviate from the inventory reference interval or to avoid changing the production rate rapidly. The buffer tanks should be used to handle unexpected production disturbances. The penalty parameters for the optimization are thus chosen as

$$\alpha_i = 10, \quad \beta_i = 1, \quad \gamma_i = 100, \quad \delta_i = 1, \quad \varepsilon_i = 0.1, \qquad i \in A$$

The optimization problem is given by (13), with the parameter values stated in this section. The problem consists of 1080 continuous variables and 360 binary variables.

5.3 Simulation

In this section, the production scheduling for one month is illustrated. An initial plan for the sales of the six products is given by the placed orders. The example is constructed as five periods with different levels of daily orders. The daily orders of each product in these periods are summarized in Table 3.

Table 3: Orders of each product in kg/day.

Days ➔	1–4	5–10	11–15	16–23	24–29
Product 1	4.80	5.53	5.96	4.80	4.11
Product 2	7.20	7.93	7.20	7.20	6.74
Product 3	0	0	0	0	0
Product 4	2.40	2.40	1.20	2.40	2.35
Product 5	4.80	2.40	4.80	4.80	4.40
Product 6	4.80	2.40	4.80	4.80	4.18

The production scheduling is run in receding horizon, such that the optimization problem (14) is solved at the beginning of each day of the month. The production scheduling gets updates on the actual production and inventory usage each day from the detailed production scheduling (lower level in the scheduling hierarchy). This information is used to update the initial conditions to the next optimization problem to be solved. In this example, the detailed production scheduling is only run when there are disturbances in production. Otherwise, it is assumed that the daily production is performed precisely according to the plan given by the production scheduling. Three utility disturbances that affect the actual daily production have been simulated: An electricity disturbance at day 2, a middle-pressure steam failure at day 12, and a high-pressure steam disturbance at day 18. The electricity

disturbance affects all areas, the middle-pressure steam disturbance affects area 2, 4, and 6, and the high-pressure steam disturbance area 1 and 3. Other areas may also be affected indirectly since the areas are connected by the flow of products at the site. In Figure 5-7, the resulting trajectories for the production rates, sales, and inventories are shown. In the figures, the plan obtained from the scheduling is marked as blue solid lines, and the actual production/sales/inventory usage is marked with dashed green lines.

Figure 6 shows that the production scheduling eliminates the backlog of products due to disturbances when this is possible, i.e., produces more than the ordered quantities at some time instances. At the end of the month, 100 % of the total ordered amount of each of the products has been delivered. Some orders are delivered one or a few days late. If this should be allowed or not depends on the contracts and relations with the customers at the actual site. In Figure 7 it can be seen that the buffer tanks are utilized to fulfill the orders, at times where there have been disturbances in production. When it is possible, the buffer tank levels are returned to within the reference interval.
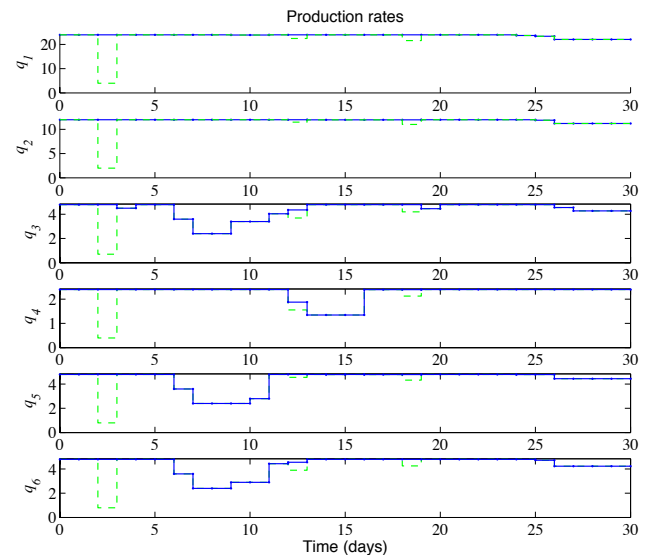

Figure 5: Production rate trajectories. The blue solid lines mark the production plan and the green dashed lines the actual production.
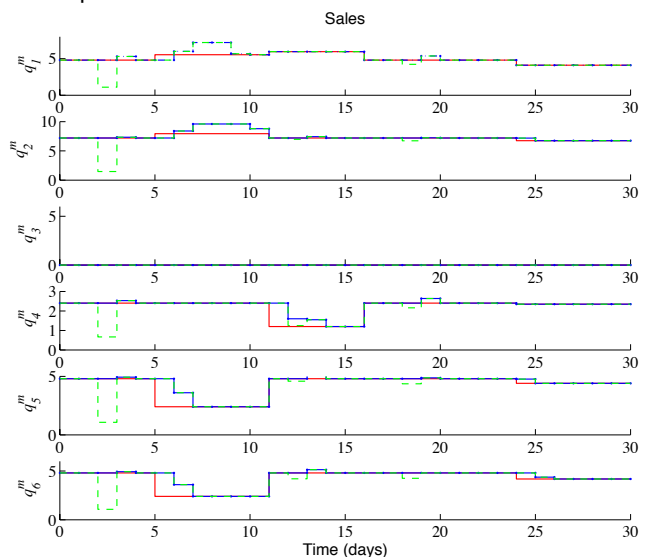

Figure 6: Sales trajectories. The red solid lines mark the orders, the blue solid lines the planned sales, and the green dashed lines the actual sales.
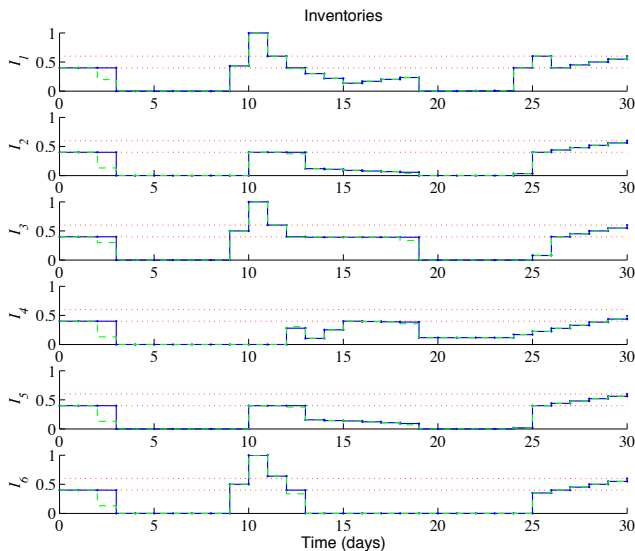
Figure 7: Inventory trajectories. The blue solid lines mark the planned inventory usage, and the green dashed lines the actual usage. The reference intervals for the buffer tanks are marked with red dotted lines.

6 CONCLUSIONS AND FUTURE WORK

An optimization model for production scheduling was introduced given specifications of functions and demands from a process industrial company. The optimization problem becomes a mixed-integer linear program that is solved every day of the month in receding horizon fashion, given updates on the actual production each day and the incoming orders. An example is given to show how the scheduling could operate during a month with some production disruptions.

It is possible to extend the model in various directions, to capture even more realistic conditions. For example, one could introduce a set of discrete production levels for each area, since it is not possible in practice to make minor changes in the production rate. Another extension is to introduce variables for individual orders of each product each day, not just an accumulated daily requested amount for each product. Such extensions are currently ongoing work.

7 ACKNOWLEDGMENTS

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