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Stochastic processes with stationary increments in time and space

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A non-negative stochastic process $(X_t)_{t\geqq 0}$ with increasing paths is said to have stationary increments in time and space if (i) the law of $X_{t+h}-X_t$ depends on h alone and (ii) the law of $\tau_{x+y}-\tau_x$ depends on y alone, where $\tau_z = \inf\{t\geqq 0: X_t \geqq z\}$. It is shown that certain subordinators, when equipped with a suitable initial distribution, have stationary increments in time and space. The class of subordinators for which this is possible is determined and the initial distribution is found in terms of the drift and the Lévy measure of the subordinator.

On Markov chains associated to random systems with complete connection

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Let $K=[\alpha, \beta]$ be a compact interval. Let f, g and q be measurable functions on K such that $f, g: K \rightarrow K$ and $q \rightarrow [0, 1]$. Define a transition probability function P from K to K by

$$P(x, E) = q(x)I_E(f(x)) + (1-q(x))I_E(g(x))$$

where I_E denotes the indicator function of E. Let $\{X_n(x), n=0, 1, \cdots\}$ denote the Markov chain induced by P and starting at x, let $\mu_{n,x}$ denote the probability measure of $X_n(x)$ and let u denote a real-valued function on K.

Theorem. Suppose that

(A1) *f and g are non-decreasing,*
(A2) *$f(x)<x<g(x)$ for $\alpha<x<\beta$,*
(A3) *$f(\beta)<\beta$ and $g(\alpha)>\alpha$,*

(A4) *f is right-continuous and g is left-continuous,*
(B1) *q is non-increasing,*
(B2) *$q(\alpha) < 1$ and $q(\beta) > 0$.*

Then

(a) *there exists a unique limit measure μ such that for all x $\mu_{n,x} \to \mu$ weakly as $n \to \infty$;*

(b) *if u is a continuous function then for all $x \in K$*

$$\lim_{N\to\infty} N^{-1} \sum_{1}^{N} u(X_n(x)) = \int u(y)\mu(dy) \quad \text{a.s.}$$

If furthermore

(C) *f, g, q and u are differentiable*

then

(c) *$N^{-1/2} \sum_1^N u(X_n(x))$ is asymptotically normally distributed.*

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Knotting of Brownian motion in 3-space

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It is shown that Brownian motion in 3-space, B, ties itself in infinitely complex knots in each small period of time. The self-intersection of B, shown in [1], means that the notion of knot must be generalised before one can make sense of the knotting of B. B is said to be *implicated* in a knot if part of it lies within a tubular neighbourhood of an open-ended knot and if it satisfies certain conditions preventing the rest of the path from 'untying' the knot. Most of the work arises from the necessity of showing that B does not untie all of the knots that it forms.

Reference

[1] DVORETSKY, A., ERDŐS, P. AND KAKUTANI, S. (1950) Double points of Brownian motion in n-space. *Acta Sci. Mat. (Szeged)* **12,** 75–81.