ERRATA

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This is an Errata for the thesis: **Topics in Potential Theory: Quadrature Domains, Balayage and Harmonic Measure.**

1. Introduction

p5, line 4 from below: should be $\leq$ for real-valued functions/measures and $=$ in the complex case.
p12, line 11 from below: Ω_t should be $m\lfloor\Omega_t$.
p13, line 3 from above: $\omega = \mathrm{Bal}(\mu/2, 1)$ should be replaced by taking ω to be the set s.t. $m\lfloor\omega = \mathrm{Bal}(\mu/2, 1)$ (or to be precise $\omega := \{U^{\mu/2} > U^{\mathrm{Bal}(\mu/2,1)}\}$).

2. paper A

p253, in Remark: $G^\mu \geq 0$ should be $G^\mu > 0$.

3. paper B

p4, line 10 from below: $\mathbb{C} \setminus \mathrm{supp}(K)$ should be $\mathbb{C} \setminus K$.
p9, in proof of Lemma 3.6: Definition of η_ε misses the exponent 2 on $|x|$.
p11, line 9 from below: should be $P(z, S(z)) = 1$ on $\partial\Omega$.
p12, line 12 from above: should be $-i\frac{2ab}{c^2}\sqrt{c^2 - x^2}$.
p13, line 11 from above: should be $\mathbb{C} \setminus \mathrm{supp}(\eta)$ instead of $\mathbb{C}$.
p14, line 9 and 11 from above: the exponent $1/2$ is missing in the last integral on $|(x + iy_0)^2 - c^2|$.
p15, in Theorem 5.1 (2): should be $\mathrm{supp}(\eta) \subset \partial D$.

4. paper C

p4, lines 5-10 from below: should be $\overline{\Omega}$ instead of Ω troughout.

5. paper D

p2, in Lemma 2.1: the sentence "Furthermore $f = 1$ on Ω, $f = \mu$ on $\overline{\Omega}^c$" should be put last in the lemma for clarity.
p6, in Remark 2: in the second line from below propert should be property.

6. paper E

p13, line 3 from below: should be $\overline{H}^h_{\chi_{K^c}}$ instead of $\overline{H}^h_{\chi_O}$.
p23, the proof of Theorem 3.4 is incorrect and should be replaced by the following one:

Proof. We start by noticing that since

$$ {}^h\hat{R}_1^F = \hat{R}_h^F/h $$

it follows that ${}^h\hat{R}_1^F$ is an $h-$potential if and only if $\hat{R}_h^F$ is a potential. For the first direction we now assume that ${}^h\hat{R}_1^F$ is an $h-$potential. If $\Omega_i \nearrow \Omega$, then

$$ \hat{R}_h^{F\setminus\Omega_i} \searrow 0 $$

(since a subsequence converges to a harmonic minorant of $\hat{R}_h^F$). Let $a \in \Omega \setminus F$ be fixed, and for each $i \geq 1$ choose Ω_i with

$$ \hat{R}_h^{F\setminus\Omega_i}(a) < 1/2^i, $$

such that $\Omega_i \nearrow \Omega$ and put $\Omega_0 = \emptyset$. Define

$$ F_i := \left(F \cap \overline{\Omega_{i+1}}\right) \setminus \Omega_i, $$

which are compact subsets of Ω.

Recall that if K_i is a sequence of decreasing compacts with intersection K, then

$$ R_h^{K_i} \searrow R_h^K, $$

so therefore it is easy to see that we for each $i \geq 1$ can choose a compact K_i with the following properties:

(1) $F_i \subset K_i \subset \Omega \setminus \{a\}$,
(2) each point of Ω belongs to at most a finite number of the K_i,
(3) each point of $\partial^E(\Omega \setminus K_i) \cap \Omega$ is regular w.r.t. $\Omega \setminus K_i$,
(4) $R_h^{K_i}(a) - R_h^{F_i}(a) < 1/2^i$.

(Also note that $R_h^{K_i}(a) = \hat{R}_h^{K_i}(a)$ for instance.) Now define

$$ u := \sum_{i=1}^{\infty} \hat{R}_h^{K_i}. $$

Since $u(a) < \infty$ we see that $u \in UP(\Omega)$, and since given $B \subset\subset \Omega$ we have that there is a constant M such that $i > M \Rightarrow K_i \cap \overline{B} = \emptyset$ we get that

$$ u = \sum_{i=1}^{M} \hat{R}_h^{K_i} + \sum_{i=M+1}^{\infty} \hat{R}_h^{K_i}, $$

where the first sum is a finite sum of continuous potentials and the second is harmonic on B, so it follows that u is continuous in B, and since B was arbitrary it follows that $u \in C(\Omega)$.

We will now prove that u is a potential. To do this we only need to note that if $k \in HP(\Omega)$ satisfies $k \leq u$, then

$$ k \leq \sum_{i=1}^{\infty} \hat{R}_h^{K_i} \Leftrightarrow k - \sum_{i=M}^{\infty} \hat{R}_h^{K_i} \leq \sum_{i=1}^{M-1} \hat{R}_h^{K_i}, $$

and since (in the last expression) the right hand side is a potential and the left hand side is subharmonic it follows that

$$ k \leq \sum_{i=M}^{\infty} \hat{R}_h^{K_i} \text{ for each } M \geq 1, $$

so $k \equiv 0$. This proves that u is a potential

To finish the proof of the first direction we define

$$ u_h := u/h $$

which is a continuous $h-$potential which is ≥ 1 on F. By definition we have that u_h has a unique continuous extension to $\Omega \cup \partial^{W(h)}\Omega$ (which we still denote u_h), and if we had

$$\overline{F} \cap \Gamma_{W(h)} \neq \emptyset,$$

then we would have that the open set (relative to $\partial^{W(h)}\Omega$)

$$\left\{x \in \partial^{W(h)}\Omega : u_h(x) > 0\right\}$$

intersects $\Gamma_{W(h)}$, and hence it has positive harmonic measure which gives a contradiction.

For the other direction suppose instead that $K := \overline{F} \cap \partial^{W(h)}\Omega \subset \Lambda^h_{W(h)}$. Since K is a compact subset of $\Lambda^h_{W(h)}$ we know that there is a $h-$potential v with limit ∞ on K. Let $K' := \{x \in F : v(x) \leq 1\}$. Then we get

$$^h\hat{R}_1^F \leq {}^h\hat{R}_1^{K'} + v,$$

and hence it is a $h-$potential. □

p24, line 2 from above: should be $A \subset U' \cap \Omega$.
p28, line 5 from above: should be $\overline{\Omega \cap \partial^E \omega}$ instead of $\overline{\Omega \cap \partial^E \Omega}$.
p44, line 8 from below: should be $\partial^2\Omega$ instead of $\partial^1\Omega$.
p47, line 6 from below + p48, in Remark + p49, Theorem 6.5: should be ω_1, ω_2 disjoint throughout.

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