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ON THE OCCASION OF THE 70th ANNIVERSARY OF BIRTHDAY OF VLADIMIR GILELEVICH MAZ'YA

Vladimir Gilelevich Maz'ya (On the Occasion of His 70th Anniversary)

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On December 31, 2007, a prominent mathematician and author of many fundamental results in the theory of functions and mathematical physics Vladimir Gilelevich Maz'ya celebrated the 70th anniversary of his birthday.

V.G. Maz'ya graduated from the Department of Mathematics and Mechanics of Leningrad State University in 1960. His first scientific paper was published as early as in 1959, when he was still a fourth year student. The same year he gave a talk at the seminar conducted by V.I. Smirnov, where he presented necessary and sufficient conditions for the validity of Sobolev-type integral inequalities. In 1962 in Moscow State University V.G. Maz'ya defended his candidate dissertation entitled "Classes of sets and embedding theorems for function spaces," in which he developed the ideas proposed in his talk at Smirnov's seminar.

Neither his diploma paper, nor his candidate dissertation was officially supervised. However, yet in his student years, he met S.G. Mikhlin and that acquaintance has much influenced his mathematical way of thinking.

From 1961 till 1986, V.G. Maz'ya worked at the Research Institute of Mathematics and Mechanics of Leningrad State University (RIMM LSU) at the position of senior researcher. In 1965 in Leningrad State University he defended his doctoral dissertation entitled "Dirichlet and Neumann problems in a domain with a nonregular boundary." From 1968 to 1978, he gave lectures to students of the Leningrad Shipbuilding Institute (now State Marine Technical University of St. Petersburg), where he was awarded the title of professor in 1976. In 1986, he transferred to the Leningrad Section of the Blagonravov Research Institute of Mechanical Engineering of the USSR Academy of Sciences, where he headed the Laboratory of mathematical models in mechanics. At the same time he headed the Mathematical Competence Center for Engineers, which had been created by V.G. Maz'ya himself. In addition, in 1980s he was among the moderators of the Leningrad all-city seminar in hydrodynamics.

In 1990, V.G. Maz'ya moved to Linkøping (Sweden) and started working at the position of professor at the local university. The same year he was conferred the degree of Honorary Doctor (Dr. E. h.) of the University of Rostock. In 1999 V.G. Maz'ya received the Humboldt prize. In 2000, he was elected Fellow of the Royal Society of Edinburgh and, in 2002, Member of the Royal Swedish Academy of Sciences.

V.G. Maz'ya was invited speaker at the International Congress of Mathematicians held in Beijing in 2002. He is on the Editorial Board of several mathematical journals published in USA, Netherlands, Germany, Sweden, India, and France. In recent years, he became professor of the University of Liverpool (Great Britain) and The Ohio State University (Columbus, Ohio, USA).

The scientific activity of V.G. Maz'ya lasts for fifty years. During this period of time, he published over 20 books and 420 scientific papers, which speaks for his remarkably high performance. Below, we briefly characterize some of these works.

Isoperimetric and integral inequalities. Yet being a student, V.G. Maz'ya discovered that Sobolev-type inequalities are equivalent to isoperimetric and isocapacitary inequalities between subsets of the domain of the function. These results were published in 1960–1961; later they were included into his candidate dissertation. The proof technique also made it possible to find exact constants in integral inequalities. In 1966, V.G. Maz'ya noticed that his proof used no specific properties of the Euclidean space and, hence, could be carried over to Riemannian manifolds. Capacitary criteria of integral inequalities are based on the so-called strong capacitary inequality, which he derived in 1964 and 1972 and recently generalized (2005 and 2006). He also discovered in 2003 that embeddings in fractional Besov spaces or Riesz potentials are equivalent to isoperimetric inequalities of the new type. It should be noted that works written by Vladimir Gilelevich in his student years have significantly influenced the development of the theory of Sobolev spaces. Moreover,

methods used in those papers have still been successfully applied, for instance, in the investigation of Sobolev spaces on metric spaces.

Nonlinear potentials. In 1970 V.G. Maz'ya and V.P. Khavin introduced the so-called nonlinear potentials and studied their properties. At the present time, the theory of nonlinear potentials, which can be regarded a generalization of the classical linear theory, is a vast field of study. It has given answers to important questions in the theory of functions, especially problems concerning the so-called exceptional sets.

Estimates for general differential operators. In the 1970s, V.G. Maz'ya and I.V. Gel'man investigated various inequalities for differential and pseudo-differential operators in a half-space. They obtained necessary and sufficient conditions for the validity of such inequalities in the case where the operators considered do not *a priori* belong to any certain type. The monograph [1] contains some final results on this subject.

Counterexamples to Hilbert's 19th problem. According to Hilbert's conjecture, solutions to first-order regular variational problems with analytic coefficients are analytic. By the middle of the 20th century this fact was established in sufficient generality and it was naturally expected that the same is true for variational problems of higher order. However, in 1968 V.G. Maz'ya showed that this is not the case. He constructed higher-order quasilinear elliptic equations with analytic coefficients whose solutions are nonsmooth.

Boundary behavior of solutions to boundary value problems. In 1970 V.G. Maz'ya obtained a sufficient condition for the regularity (in the sense of Wiener) of a boundary point for a certain class of quasilinear second-order elliptic equations containing the p-Laplacian. However, until recently, nothing could be said about the regularity of a boundary point for equations of order higher than two. In 2002 V.G. Maz'ya generalized the Wiener test to high-order elliptic equations. This result was the topic of his report at the International Congress of Mathematicians in Beijing.

Boundary integral equations. In 1967, Yu.D. Burago and V.G. Maz'ya investigated single-layer and double-layer harmonic potentials in the space C for a wide class of surfaces. In 1981 V.G. Maz'ya proposed a new method for studying boundary value integral problems. The method was based on the preliminary investigation of a certain auxiliary boundary value problem. This approach was successfully used to establish some theorems on solvability of classical boundary value integral equations on piecewise smooth surfaces (in 2005, V.G. Maz'ya and T.O. Shaposhnikova generalized this method to the case of Lipschitz surfaces). V.G. Maz'ya and A.A. Solov'ev were first to consider integral equations on curves with peaks (1990). Using the aforementioned approach, they developed the theory of logarithmic potential for equations of the elasticity theory in a plane domain with a peak (2001).

Sobolev spaces in singularly perturbed domains. A singularly perturbed domain is a domain which depends on small or large parameters in such a way that the limiting domain degenerates. Properties of Sobolev spaces in such domains significantly depend on singular parameters. The influence of singular parameters on the norm of the operators of embedding, extension, and boundary trace has been studied by V.G. Maz'ya and S. V. Poborchii since 1980s. Results of this kind are useful in substantiating the asymptotic behavior of the solutions to boundary value problems in singularly perturbed domains. The theory of Sobolev spaces in such domains is presented in the recently published book [2].

The asymptotic theory of elliptic boundary value problems in singularly perturbed domains. In the two-volume monograph [3], V.G. Maz'ya, S.A. Nazarov, and B.A. Plamenevskii investigate the asymptotic behavior of the solutions to elliptic boundary value problems in the case where the boundary of the domain has small singular perturbations. The same range of problems related to the asymptotic theory of multistructures is studied in the monograph [4] written by V.G. Maz'ya in co-authorship with V.A. Kozlov and A.B. Movchan.

Hadamard's scientific biography. V.G. Maz'ya and his wife T.O. Shaposhnikova made a remarkable contribution to the history of mathematics: they wrote a book about life and work of the great French mathematician Jacques Hadamard [5, 6]. The book contains volumetric material collected by the authors in various archives and libraries and presents an analysis of Hadamard's huge scientific heritage. In 2003, the authors of this book received the Verdaguer Prize of the French academy of Sciences.

Other areas in which Vladimir Gilelevich has distinguishable works include approximation by analytic and harmonic functions, the problem with directional derivative, degenerating elliptic and pseudo-differential operators, uniqueness theorems for Lamé systems with data on a part of the boundary, methods for solution of ill-posed boundary value problems, spectral properties of the Schrödinger operator, multiplicators in pairs of spaces of smooth functions, numerical methods for solution of differential equations, water waves, and singularities of the solutions to nonlinear differential equations. This list can be continued.

We wish Vladimir Gilelevich good health and further achievements.

M.V. Anolik, Yu.D. Burago, Yu.K. Dem'yanovich, S.V. Kislyakov, V.P. Khavin, G.A. Leonov, N.F. Morozov, S.V. Poborchii, N.N. Ural'tseva, N.A. Shirokov.

REFERENCES

1. I. V. Gel'man and V. G. Maz'ya, *Abschätzungen für Differentialoperatoren im Halbraum* (Akademie, Berlin, 1981; Birkhäuser, Basel–Boston, 1982).
2. V. G. Maz'ya and S. V. Poborchii, *Theorems of Embedding and Extension for Functions in Non-Lipschitz Domains* (St. Petersb. Univ., St. Petersburg, 2006).
3. V. G. Maz'ya, S. A. Nazarov, and B. A. Plamenevskii, *Asymptotische Theorie elliptischer Randwertaufgaben in singulär gestörten Gebieten*, B. I, Störungen isolierter Randsingularitäten, B. II, Nichtlocale Störungen (Akademie-Verlag, Berlin, 1991); English transl.: *Asymptotic Theory of Elliptic Boundary Value Problems in Singularly Perturbed Domains,* Vol. 1-2, Operator Theory, Advances and Applications. Vol. 111, 112 (Birkhäuser, Basel–Boston–Berlin, 2000).
4. V. A. Kozlov, V. G. Maz'ya, and A. B. Movchan, *Asymptotic Analysis of Fields in Multistructures* (Oxford Science Publications, Oxford, 1999).
5. V. G. Maz'ya and T. O. Shaposhnikova, *Jacques Hadamard, a Universal Mathematician, History of Mathematics*, 14. AMS, Providence, RI (London Mathem. Soc., London, 1998).
6. V. G. Maz'ya and T. O. Shaposhnikova, *Jacques Hadamard, un Mathématicien Universel* (EDP Sciences, 2005).