New Local Conditions for a Graph to be Hamiltonian*

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Abstract. For a vertex w of a graph G the ball of radius 2 centered at w is the subgraph of G induced by the set $M_2(w)$ of all vertices whose distance from w does not exceed 2. We prove the following theorem: Let G be a connected graph where every ball of radius 2 is 2-connected and $d(u) + d(v) \geq |M_2(w)| - 1$ for every induced path uwv. Then either G is hamiltonian or $K_{p,p+1} \subseteq G \subseteq K_p \vee \overline{K_{p+1}}$ for some $p \geq 2$ where $\vee$ denotes join.

As a corollary we obtain the following local analogue of a theorem of Nash-Williams: A connected r-regular graph G is hamiltonian if every ball of radius 2 is 2-connected and $r \geq \frac{1}{2}(|M_2(w)| - 1)$ for each vertex w of G.

Keyword: Hamilton cycle, local condition, ball

1. Introduction

In [3-5] the author and N.K. Khachatryan developed some local criteria for the existence of Hamilton cycles in a connected graph, which are analogues of the global criteria due to Dirac [11], Ore [16] and others. The idea was to show that the global concept of hamiltonicity can, under rather general conditions, be captured by local phenomena, using the structure of balls of small radii. In this paper we present a new result on this topic.

We use [10] for terminology and notation not defined here and consider finite simple graphs only. Let $V(G)$ and $E(G)$ denote, respectively, the

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vertex set and edge set of a graph G, and let $d(u,v)$ denote the distance between vertices u and v in G. For each vertex u of G and a positive integer r, we denote by $N(u)$ and $M_r(u)$ the sets of all $v \in V(G)$ with $d(u,v) = 1$ and $d(u,v) \leq r$, respectively. For a vertex u of a graph G the ball $G_r(u)$ of radius r centered at u is the subgraph of G induced by the set $M_r(u)$. The square of a graph G is the graph obtained from G by joining every pair of vertices at distance 2 in G. A graph G is claw-free if G has no induced subgraph isomorphic to $K_{1,3}$. A graph G is called *1-tough* if for every nonempty set $S \subset V(G)$, the graph $G - S$ has at most $|S|$ components. Clearly, every hamiltonian graph is 1-tough.

A classical result on hamiltonian graphs is the following theorem.

Theorem A (Ore [16]). Let G be a graph on at least 3 vertices such that $d(u) + d(v) \geq |V(G)|$ for each pair of nonadjacent vertices u, v. Then G is hamiltonian.

Later, several authors (see, for example [12,14]) have found that the condition in Theorem A can be relaxed if we allow a class of exceptions. Set

$$\mathcal{K}=\{G : K_{p,p+1} \subseteq G \subseteq K_p \vee \overline{K_{p+1}} \text{ for some } p \geq 2\} \ (\vee \text{ denotes join}).$$

Theorem B (Jung [12], Nara [14]). Let G be a 2-connected graph such that $d(u) + d(v) \geq |V(G)| - 1$ for each pair of nonadjacent vertices u, v. Then either G is hamiltonian or $G \in \mathcal{K}$.

The next result shows that for regular graphs the condition of Theorem B is sufficient for hamiltonicity without any exceptions.

Theorem C (Nash-Williams [15]). A 2-connected regular graph G is hamiltonian if $d(u) \geq \frac{1}{2}(|V(G)| - 1)$ for each vertex u.

The author and Khachatryan [5] improved Theorem A to a result of local nature. We proved that a connected graph G with $|V(G)| \geq 3$ is hamiltonian if $d(u) + d(v) \geq |N(u) \cup N(v) \cup N(w)|$ for each path uwv with $uv \notin E(G)$. This result was extended in the following way:

Theorem D (Asratian et al. [7]). Let G be a connected graph with $|V(G)| \geq 3$ where

(1.1) $d(u) + d(v) \geq |N(u) \cup N(w) \cup N(v)| - 1$, for each path uwv with $uv \notin E(G)$, and

(1.2) $d(u,v) = 2 \Longrightarrow |N(u) \cap N(v)| \geq 2$.

Then either G is hamiltonian or $G \in \mathcal{K}$.

Note that all claw-free graphs satisfy condition (1.1).

If $w \in N(u) \cap N(v)$ then $N(u) \cup N(w) \cup N(v) \subseteq M_2(w)$. Therefore Theorem D implies the following corollaries:

Corollary D1. Let G be a connected graph with $|V(G)| \geq 3$ where $d(u) + d(v) \geq |M_2(w)| - 1$ for each path uwv with $uv \notin E(G)$. If G satisfies condition (1.2) then either G is hamiltonian or $G \in \mathcal{K}$.

Corollary D2 [4]. A connected graph G with $|V(G)| \geq 3$ is hamiltonian if $d(u) + d(v) \geq |M_2(w)|$ for each path uwv with $uv \notin E(G)$.

A. Saito [17] showed that for a 2-connected graph G of diameter 2 condition (1.2) in Theorem D can be omitted. This gives the following generalization of Theorem B:

Theorem E (Saito [17]). Let G be a 2-connected graph of diameter 2 where $d(u) + d(v) \geq |N(u) \cup N(w) \cup N(v)| - 1$ for each path uwv with $uv \notin E(G)$. Then either G is hamiltonian or $G \in \mathcal{K}$.

Some other localization results related to Theorems A, B and D can be found in [1-9,13].

Remark 1. Every graph G satisfying the conditions of any one of the above theorems has the following property: *all balls of radius 2 in G are 2-connected.* This is evident if G satisfies the conditions of any one of the Theorems A, B, C and E because then $G_2(u) = G$ for each vertex u and G is 2-connected. If G satisfies the condition (1.2) of Theorem D then it is not difficult to see that this implies that all balls of radius 2 in G are 2-connected.

2. Results

Our main result is the following theorem:

Theorem 1. Let G be a connected graph with $|V(G)| \geq 3$ where
(2.1) $d(u) + d(v) \geq |M_2(w)| - 1$ for every path uwv with $uv \notin E(G)$, and
(2.2) all balls of radius 2 in G are 2-connected.
Then either G is a hamiltonian graph or $G \in \mathcal{K}$.

It follows from Remark 1 that Theorem 1 is a joint generalization of Theorems A, B and C as well as Corollaries D1 and D2. Since the set $\mathcal{K}$ contains no regular graphs, Theorem 1 implies the following corollary:

Corollary 2. Let G be a connected regular graph on at least 3 vertices where every ball of radius 2 is 2-connected and $d(u) \geq \frac{1}{2}(|M_2(u)| - 1)$, for each vertex u. Then G is a hamiltonian graph.

Corollary 2 is a generalization of Theorem C because for each graph G satisfying Theorem C we have $G_2(u) = G$ for each vertex u. The next result follows from Corollary 2:

Corollary 3. Let G be a connected regular graph on at least 3 vertices where $d(u) \geq \frac{1}{2}|M_2(u)|$, for each vertex u. Then G is a hamiltonian graph.

A similar condition for hamiltonicity of an arbitrary graph by using balls of radius 3 was obtained in [5]: A connected graph G on at least 3 vertices is hamiltonian if $d(u) \geq \frac{1}{2}|M_3(u)|$ for each vertex u of G. Note that this result also follows from Theorem 1.

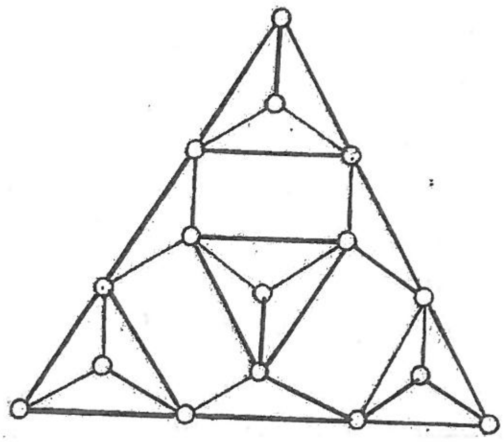
Now we discuss the sharpness and usefulness of Theorem 1.

Every graph G satisfying the conditions for any one of the Theorems A, B, and C is dense ($|E(G)| \geq \textit{constant} \cdot |V(G)|^2$) and has diameter 2. In contrast with this, for every integer $n \geq 2$ there are sparse graphs of diameter n which satisfy the conditions of Theorem 1. For example, the graph $G = P_{2n}^2$ which is the square of a path P_{2n} of length $2n$ satisfies the conditions of Theorem 1 and has $4n - 3$ edges and diameter n.

Similarly for every $n \geq 2$, the square of a cycle of length $4n$ is a 4-regular graph of diameter n which satisfies the conditions of Corollary 2 and has $8n$ edges.

Remark 2. The class of graphs satisfying the conditions of Theorem 1 contains some claw-free graphs (for example, P_{2n}^2) as well as graphs which are not claw-free (for example, $K_{n,n}$ for $n \geq 3$). The graph P_{2n}^2, $n \geq 3$ does not satisfy Theorem E and condition (1.2) of Theorem D. Thus Theorem 1 is incomparable to Theorems D and E in the sense that neither theorem implies the other. Furthermore Theorem 1 is incomparable to any theorem which gives a sufficient condition for a claw-free graph to be hamiltonian.

Remark 3. Theorem 1 cannot be relaxed by replacing the condition (2.1) by the condition (1.1). Consider, for example, the graph G below.



Since G is claw-free, G satisfies condition (1.1). Furthermore every ball of radius 2 in G is 2-connected, that is, G satisfies condition (2.2). However it is not difficult to verify that G is not hamiltonian.

Remark 4. Condition (2.2) for a graph G implies that G is 2-connected. Theorem 1 cannot be relaxed by replacing the condition (2.2) by the condition G is 2-connected. Consider, for example, the graph G constructed by N.K. Khachatryan where $V(G) = \{u_1, u_2, ..., u_n, v_1, v_2, ..., v_n, x, y, z\}$, $n \geq 6$

$$\text{and } E(G) = \{u_i u_j, v_i v_j : 1 \leq i \leq n-3, i < j \leq n\} \cup \{u_{n-2}x, v_{n-2}x, u_{n-1}y, v_{n-1}y, u_n z, v_n z\}.$$

Clearly, the graph G is 2-connected and satisfies condition (2.1). However G is not hamiltonian.

3. Proofs

Let C be a cycle of G. We denote by $\overrightarrow{C}$ the cycle C with a given orientation, and by $\overleftarrow{C}$ the cycle C with the reverse orientation. We use u^+ to denote the successor of u on $\overrightarrow{C}$ and u^- to denote its predecessor. If $u, v \in V(C)$, then $u\overrightarrow{C}v$ denotes the consecutive vertices of C from u to v in the direction specified by $\overrightarrow{C}$. The same vertices, in reverse order, are given by $v\overleftarrow{C}u$. If $u = v$ then $u\overrightarrow{C}v$ and $v\overleftarrow{C}u$ consists of one vertex u.

It was proved in [7] that if a 2-connected graph G satisfies condition (1.1), then either $G \in \mathcal{K}$ or G is 1-tough. Since $N(u) \cup N(v) \cup N(w) \subseteq M_2(w)$, this implies the following property:

Lemma 1. Let G be a 2-connected graph where $d(u)+d(v) \geq |M_2(w)|-1$ for every path uwv with $uv \notin E(G)$. Then either $G \in \mathcal{K}$ or G is 1-tough .

Remark 5. Let uwv be a path in G with $uv \notin E(G)$. Since $d(u)+d(v) = |N(u) \cap N(v)| + |N(u) \cup N(v)|$, the condition $d(u)+d(v) \geq |M_2(w)|-1$ is equivalent to the condition $|N(u) \cap N(v)| \geq |M_2(w) - (N(u) \cup N(v))| - 1$.

Lemma 2. Let G be a non-hamiltonian connected graph where $d(x) + d(y) \geq |M_2(w)| - 1$ for each path xwy with $xy \notin E(G)$. Furthermore, let $\overrightarrow{C}$ be a longest cycle of G , and v, w be vertices such that $v \in V(G) - V(\overrightarrow{C})$, $w \in N(v) \cap V(\overrightarrow{C})$ and

$$N(v) \cap N(w^+) = N(v) \cap N(w^-) = \{w\}.$$

Then w^+ is adjacent to every vertex in $M_2(w) - (M_1(v) \cup \{w^+\})$ and w^- is adjacent to every vertex in $M_2(w) - (M_1(v) \cup \{w^-\})$. In particular, $w^+w^- \in E(G)$.

Proof. We have $v, w^+ \in M_2(v)$, $vw^+ \notin E(G)$ and $d(v) + d(w^+) \geq |M_2(w)| - 1$. By Remark 5 this is equivalent to

$$|N(v) \cap N(w^+)| \geq |M_2(w) - (N(v) \cup N(w^+))| - 1.$$

Then $N(v) \cap N(w^+) = \{w\}$ implies $M_2(w) - (N(v) \cup N(w^+)) = \{v, w^+\}$. This means that w^+ is adjacent to every vertex in $M_2(w) - (M_1(v) \cup \{w^+\})$. In particular, we get $w^+w^- \in E(G)$.

In a similar way we can show that w^- is adjacent to every vertex in $M_2(w) - (M_1(v) \cup \{w^-\})$.

Proof of Theorem 1. Let G be a graph satisfying the conditions in the theorem. Suppose that G is not hamiltonian. Choose a longest cycle C in G and specify an orientation of C.

Claim 1. There is a vertex in $V(G) - V(C)$ which has at least two neighbors on C.

Proof. Suppose that the claim is false, and consider a vertex $v \in V(G) - V(C)$ with a neighbor on C. Let $N(v) \cap V(C) = \{w\}$. Since the ball $G_2(w)$ is 2-connected, there is a path P in $G_2(w) - \{w\}$ with origin w^+ and terminus v. Clearly, P has an internal vertex which lies on $\overrightarrow{C}$ since otherwise deleting from C the edges ww^+ and adding P we could obtain a cycle that is longer than C. Let z be a vertex on $V(P) \cap V(C)$ such that all other vertices on the path $z\overrightarrow{P}v$ do not belong to C. Clearly, $z \neq w^+$

and $z \neq w^{++}$ because otherwise there is a cycle which is longer than C. By Lemma 2, $w^-w^+, z^-z^+ \in E(G)$ and also $w^-z \in E(G)$ since $z \in M_2(w) - (M_1(v) \cup \{w^-\})$. But then the cycle $w^-z\overrightarrow{P}vw\overrightarrow{C}z^-z^+\overrightarrow{C}w^-$ is longer than C, a contradiction.

Claim 2. There is a vertex $u \in V(G) - V(C)$ and a vertex $w \in N(u) \cap V(C)$ such that either $|N(u) \cap N(w^+)| \geq 2$ or $|N(u) \cap N(w^-)| \geq 2$.

Proof. By contradiction. Suppose that

$$N(u) \cap N(w^+) = N(u) \cap N(w^-) = \{w\}, \tag{1}$$

for each pair u, w where $u \in V(G) - V(C)$ and $w \in N(u) \cap V(C)$.

Choose a vertex v in $V(G) - V(C)$ such that $|N(v) \cap V(C)| \geq 2$ (see Claim 1). Set $W = N(v) \cap V(C)$ and $k = |W|$. Let $w_1, \ldots, w_k$ be the vertices of W, occuring on $\overrightarrow{C}$ in the order of their indicies. By (1), we have

$$N(v) \cap N(w_i^+) = N(v) \cap N(w_i^-) = \{w_i\}, \qquad (i = 1, \ldots, k). \tag{2}$$

By the condition of Theorem 1, the ball $G_2(v)$ is a 2-connected graph. Consider in $G_2(v) - w_1$ a shortest path $P = u_0u_1...u_t$ where $u_0 = w_1^+$ and $u_t \in \{w_2, ..., w_k\}$. By (2), $u_1 \notin N(v)$. Then $d(v, u_1) = 2$ and there is a vertex $v_1 \in N(v)$ which is adjacent to u_1. We will show that $v_1 = w_1$. Clearly, $v_1 \in N(v) \cap V(C) = \{w_1, ..., w_k\}$ because otherwise there is a cycle C' which is longer than C. (For example, $C' = w_1vv_1u_1w_1^+\overrightarrow{C}w_1$ if $u_1 \notin V(C)$ and $C' = w_1vv_1u_1w_1^+\overrightarrow{C}u_1^-u_1^+\overrightarrow{C}w_1$ if $u_1 \in V(C)$.) Suppose that $v_1 \in \{w_2, ..., w_k\}$, say $v_1 = w_2$. Clearly, $w_1^+ \in M_2(w_2) - (M_1(v) \cup \{w_2^+\})$. Therefore, by (2) and Lemma 2, w_2^+ is adjacent to w_1^+. But then the cycle $w_1vw_2\overleftarrow{C}w_1^+w_2^+\overrightarrow{C}w_1$ is longer than C, a contradiction. Therefore, $v_1 \notin \{w_2, ..., w_k\}$ and $v_1 = w_1$, that is,

$$u_1w_1 \in E(G), \quad u_1w_i \notin E(G), \qquad (i = 2, \ldots, k). \tag{3}$$

Since u_1 is adjacent to the consecutive vertices w_1 and w_1^+ on $\overrightarrow{C}$, and C is a longest cycle of G,

$$u_1 \in V(C). \tag{4}$$

We will show now that

$$vu_2 \notin E(G). \tag{5}$$

Suppose to the contrary that $vu_2 \in E(G)$. Then (3) and $w_1 \notin V(P)$ imply that $u_2 \in N(v) - V(C)$. We have $u_1u_2 \in E(G), u_1 \in V(C)$ and $u_2 \in N(v) - V(C)$. Therefore, by (1), $u_1^- u_1^+ \in E(G)$. But then the cycle $w_1 v u_2 u_1 w_1^+ \overrightarrow{C} u_1^- u_1^+ \overrightarrow{C} w_1$ is longer than C.

Thus $vu_2 \notin E(G)$. Clearly, $u_2 \in M_2(w_1)$ because $w_1u_1, u_1u_2 \in E(G)$. Then by (5) and Lemma 2, necessarily $w_1^+ u_2 \in E(G)$. But this contradicts the assumption that $u_0u_1...u_t$ is a shortest path with origin w_1^+ and terminus in $\{w_2, ..., w_k\}$.

The proof of Claim 2 is completed.

We continue to prove the theorem. By Claim 2, there is a vertex $v \in V(G) - V(C)$ and a vertex $w_1 \in V(C)$ such that either $|N(v) \cap N(w_1^+)| \geq 2$ or $|N(v) \cap N(w_1^-)| \geq 2$. Without loss of generality we assume that $|N(v) \cap N(w_1^+)| \geq 2$. The choice of C implies that $N(v) \cap N(w_1^+) \subseteq V(C)$. Set $W = N(v) \cap V(C)$ and $k = |W|$. Let $w_1, \ldots, w_k$ be the vertices of W, occurring on $\overrightarrow{C}$ in the order of their indices, $k \geq 2$.

Set $W^+ = \{w_1^+, ..., w_k^+\}$. We will count the number of edges between W^+ and W which we denote by $e(W^+, W)$. The choice of C implies that $W^+ \cup \{v\}$ is an independent set, and $N(w_i^+) \cap N(v) \cap (V(G) - V(C)) = \emptyset$, for $1 \leq i \leq k$. Moreover, for each $i, 1 \leq i \leq k$, $d(v, w_i^+) = 2$ and $w_i \in N(v) \cap N(w_i^+)$, so by the hypothesis of the theorem and by Remark 5,

$$|N(v) \cap N(w_i^+)| \geq |M_2(w_i) - (N(v) \cup N(w_i^+)| - 1. \tag{6}$$

Obviously, $N(w_i) \cap W^+ \subseteq N(w_i) - (N(v) \cup N(w_i^+) \cup \{v\})$.

Thus,

$|N(w_i) \cap W^+| \leq |N(w_i) - (N(v) \cup N(w_i^+)| - 1 \leq |M_2(w_i) - (N(v) \cup N(w_i^+))| - 1$.

This and (6) imply that $|N(w_i) \cap W^+| \leq |N(v) \cap N(w_i^+)|$.

Hence,

$e(W^+, W) = \sum_{i=1}^k |N(w_i) \cap W^+| \leq \sum_{i=1}^k |N(v) \cap N(w_i^+)| = e(W^+, W)$.

It follows, for each $i, 1 \leq i \leq k$, that

$$N(w_i) - (N(v) \cup N(w_i^+) \cup \{v\}) = N(w_i) \cap W^+ \subseteq W^+. \tag{7}$$

Noting that $k \geq 2$ and the fact that $|N(w_1^+) \cap N(v)| \geq 2$, we now prove by contradiction that $w_i^+ = w_{i+1}^-$ for each $i = 1, ..., k$. (We consider $w_{k+1} = w_1$.)

Assume without loss of generality that $w_1^+ \neq w_2^-$, whence $w_2^- \notin W^+$. Observe that $w_2^- \in N(w_2^+)$, otherwise from (7), $w_2^- \in W^+$. Since C is a longest cycle, $w_2^- w_3^- \notin E(G)$. Hence $w_2^+ \neq w_3^-$. Repetition of this argument shows that $w_i^+ \neq w_{i+1}^-$ and $w_i^+ w_i^- \in E(G)$ for all $i \in \{1, \ldots, k\}$. By assumption, $N(w_1^+) \cap N(v)$ contains a vertex $x \neq w_1$. Since C is a longest cycle, $x \in V(C)$, say that $x = w_i$. But then the cycle $w_1 v w_i w_1^+ \overrightarrow{C} w_i^- w_i^+ \overrightarrow{C} w_1$ is longer than C. This contradiction proves that $w_i^+ = w_{i+1}^-$ for each $i = 1, ..., k$.

Since C is a longest cycle, there exists no path joining two vertices of $W^+ \cup \{v\}$ with all internal vertices in $V(G) \setminus V(C)$. Hence the number of components in $G - W$ is greater than $|W|$, that is, G is not 1-tough. Then Lemma 1 implies that $G \in \mathcal{K}$. This completes the proof of Theorem 1.

Proof of Corollary 3. Let uwv be a path in G with $uv \notin E(G)$. Since G is regular, we have that $d(u) + d(v) = 2d(w) \geq |M_2(w)|$. The condition $d(u) + d(v) \geq |M_2(w)|$ and Remark 5 imply that $|N(u) \cap N(v)| \geq 2$ since $u, v \in M_2(w) - (N(u) \cup N(v))$. Thus G satisfies the condition (1.2). Then, by Remark 1, all balls of radius 2 in G are 2-connected. Therefore, by Corollary 2, G is hamiltonian.

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