

**Matrix Methods in Data Mining and Patter Recognition**  
**Computer Assignment**  
**Least Squares and Orthogonal Transformations**

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## ASSIGNMENT

An introduction to basic matrix concepts and to using orthogonal transformations and decompositions in Matlab.

## SPECIFIC TASKS

1. Solve the following least squares problems using the normal equations and QR decomposition. In both cases, plot the data and the polynomial approximation.

(a) Fit a third degree polynomial to the data

$$\begin{array}{c|cccc} t & 1 & 2 & 3 & 4 \\ \hline f(t) & 1 & 2 & 2 & 4 \end{array}$$

(b) Fit a third degree polynomial to the data

$$\begin{array}{c|ccccc} t & 1 & 2 & 3 & 4 & 5 \\ \hline f(t) & 1 & 2 & 2 & 4 & 5 \end{array}$$

Explain the differences between (a) and (b).

2. Fit the third degree polynomial  $P(t) = a_0 + a_1t + a_2t^2 + a_3t^3$  to the following data, using normal equations.

$$\begin{array}{c|cccc} t & 501 & 502 & 503 & 504 \\ \hline f(t) & 1 & 2 & 2 & 4 \end{array}$$

What is the condition number of the matrix in the normal equations? Use a better model and compute the condition number of the normal equations.

3. (a) Compute the solution of the least squares problem

$$\min_{\beta} \|X\beta - y\|, \quad X = \begin{pmatrix} 1 & 1 \\ \epsilon & 0 \\ 0 & \epsilon \end{pmatrix}, \quad y = \begin{pmatrix} 1 \\ 1 \\ 1 \end{pmatrix}$$

for the values  $\epsilon = 1, 10^{-1}, \dots, 10^{-8}, 10^{-9}$  using QR decomposition and normal equations. Compare the results. Which are more accurate?

- (b) Illustrate the condition numbers of the data matrix and the normal equations as a function of  $\epsilon$ .
4. Write a MATLAB function for reducing

$$\begin{pmatrix} R \\ x^T \end{pmatrix} \longrightarrow Q^T \begin{pmatrix} R \\ x^T \end{pmatrix} = \begin{pmatrix} \hat{R} \\ 0 \end{pmatrix},$$

where  $R$  is upper triangular, using  $n$  plane rotations. (See Section 5.6).

5. (a) Write a Matlab function `[Q,R,P]=qr piv(A)` that computes a QR decomposition with column pivoting. Construct a matrix that does not have full column rank, and apply the function to it. Also test the function on the Kahan matrix.
- (b) For reasons of further enhanced numerical stability one sometimes uses column pivoting also for problems with full column rank. Solve the least squares problem  $\min \|Ax - b\|_2$ , where  $A$  and  $b$  are taken from item 1 above.
6. (a) Is it possible to construct a finite sequence of orthogonal transformations,  $Q = Q_1 Q_2 \cdots Q_n$ , such that when applied to a symmetric matrix  $A$ , it is diagonalized,

$$A \longrightarrow Q^T A Q = \text{diag}$$

(no coding, answer and motivate).

- (b) Write a MATLAB function for reducing a non-symmetric square matrix to upper Hessenberg form

$$A \longrightarrow Q^T A Q = H,$$

using Householder transformations (how many?). See Chapter 15.

- (c) Exactly the same function can be used to reduce a symmetric matrix to tridiagonal form

$$A \longrightarrow Q^T A Q = T.$$

(Check that).