On the non-vanishing property for real analytic solutions of the p-Laplace equation

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Abstract. The main result of the present talk is motivated by the non-vanishing property for real analytic solutions to the p-Laplace equation and was inspired by the following question of John Lewis [4]: Does there exist a real homogeneous polynomial $u(x)$ of degree $m = \deg u \geq 2$ in $\mathbb{R}^n$, $n \geq 3$ satisfying

$$\Delta_p u := |Du|^2 \Delta u + \frac{p-2}{2} \langle Du, D|Du|^2 \rangle = 0, \tag{1}$$

where $p > 1$, $p \neq 2$? Lewis itself answered in negative this question in two dimensions in [4]. On the other hand, notice that for any $d \geq 2$ and $n \geq 2$ there exist plenty quasi-polynomial $C^{d,\alpha}$-smooth solutions of (1) in $\mathbb{R}^n$ [3], [1], [2], [8], [6].

We have the following particular answer on the Lewis question.

Theorem 1. *Any real homogeneous cubic polynomial solution of (1) in $\mathbb{R}^n$ for $n \geq 2$ is identically zero.*

The proof of Theorem 1 makes use of a nonassociative algebra argument developed earlier for similar problems in [7], [5].

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