Beta

You can use this list during the exam in PDE, 2008-10-13.

1. Characteristics for non-linear 1st order PDE $F(x, y, z, p, q) = 0$:

$$\frac{dx}{dt} = F'_p,\ \frac{dy}{dt} = F'_q,\ \frac{dz}{dt} = p \cdot F'_p + q \cdot F'_q$$
$$\frac{dp}{dt} = -F'_x - F'_z p, \qquad \frac{dq}{dt} = -F'_y - F'_z q.$$

and the strip condition: $\frac{d}{ds} z_0(s) = p_0(s) \cdot \frac{dx_0}{ds} + q_0(s) \cdot \frac{dy_0}{ds}$

2. Jump condition (or the Rankine-Hugoniot condition) for the inviscid Burger's equation:

$$\xi'(y) = \frac{G(u_r) - G(u_l)}{u_r - u_l}$$

3. Characteristics for the 2nd order linear PDE with principal part $au''_{xx} + bu''_{xy} + cu''_{yy}$:

$$\frac{dy}{dx} = \frac{b \pm \sqrt{b^2 - 4ac}}{2a}$$

4. Wave equation:
   - d' Alembert's formula: $u(x,t) = \frac{1}{2}[g(x+ct) + g(x-ct)] + \frac{1}{2c}\int_{x-ct}^{x+ct} h(s)\, ds$
   - Duhamel's principal for nonhomogeneous wave equation with R.H.S. f and homogeneous initial conditions: $u(x,t) = \frac{1}{2c}\int_0^t \int_{x-c(t-s)}^{x+c(t-s)} f(\xi, s)\, d\xi\, ds$
   - Kirchhoff's formula:

$$u(x,t) = \frac{1}{4\pi}\frac{\partial}{\partial t}\left(t \int_{|\xi|=1}^{t} g(x + ct\,\xi) dS_\xi\right) + \frac{t}{4\pi}\int_{|\xi|=1}^{t} h(x + ct\,\xi) dS_\xi$$

5. Laplace equation:
   - fundamental solution: $\Psi(x) = \begin{cases} \frac{1}{2\pi}\ln|x| & n = 2 \\ -\frac{1}{(n-2)\omega_n |x|^{n-2}} & n \geq 3 \end{cases}$
   - solution of Poisson's equation: $u(x) = \int_{\mathbb{R}^n} \Psi(x-y) f(y) dy$
   - solution of the Dirichlet problem in the ball $B_0(R)$:

$$u(\xi) = \frac{R^2 - |\xi|^2}{R\,\omega_n}\int_{|x|=R} \frac{g(x)}{|x-\xi|^n} dS_x$$