

Beta (Formelsamling)

You can use this list during the exam in PDE, 2009-03-13.

1. Characteristics for general 1st order PDE $F(x, y, z, p, q) = 0$:

$$\begin{aligned}\frac{dx}{dt} &= F'_p, & \frac{dy}{dt} &= F'_q, & \frac{dz}{dt} &= p \cdot F'_p + q \cdot F'_q \\ \frac{dp}{dt} &= -F'_x - F'_z p, & \frac{dq}{dt} &= -F'_y - F'_z q.\end{aligned}$$

The strip condition: $\frac{d}{ds} z_0(s) = p_0(s) \cdot \frac{dx_0}{ds} + q_0(s) \cdot \frac{dy_0}{ds}$

2. Characteristics for the 2nd order linear PDE with principal part $au''_{xx} + bu''_{xy} + cu''_{yy}$:

$$\frac{dy}{dx} = \frac{b \pm \sqrt{b^2 - 4ac}}{2a}$$

3. The wave equation:

- d'Alembert's formula: $u(x, t) = \frac{1}{2} [g(x + ct) + g(x - ct)] + \frac{1}{2c} \int_{x-ct}^{x+ct} h(s) ds$

- Kirchhoff's formula:

$$u(x, t) = \frac{1}{4\pi} \frac{\partial}{\partial t} \left(t \int_{|\xi|=1} g(x + ct \xi) dS_\xi \right) + \frac{t}{4\pi} \int_{|\xi|=1} h(x + ct \xi) dS_\xi$$

4. Laplace equation:

- The fundamental solution: $\Psi_n(x) = \begin{cases} \frac{1}{2\pi} \ln |x| & n = 2 \\ -\frac{1}{(n-2)\omega_n |x|^{n-2}} & n \geq 3 \end{cases}$

- Solution of Poisson's equation $\Delta u = f$:

$$u(x) = \int_{\mathbb{R}^n} \Psi(x - y) f(y) dy$$

- the Green function $G(x, y)$ of an open domain Ω is the harmonic (except for the diagonal $x = y$) function in each variable such that $G(x, y) - \Psi(x - y)$ is continuous in Ω and $G(\cdot, y)|_{\partial\Omega} \equiv 0$.

5. The heat equation:

- Solution of the pure initial problem $\Delta u = u_t$, $u(x, 0) = g(x)$:

$$u(x, t) = \frac{1}{(4\pi t)^{\frac{n}{2}}} \int_{\mathbb{R}^n} e^{-\frac{|x-y|^2}{4t}} g(y) dy$$

6. The area ω_n of the unit sphere in \mathbb{R}^n :

$$\omega_n = \frac{2\pi^{\frac{n}{2}}}{\Gamma\left(\frac{n}{2}\right)}$$