Siteswap state diagrams

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October 4, 2004

This document contains diagrams that can be used for quickly finding valid "siteswap" juggling patterns and transitions between different such patterns. These diagrams contain all possible siteswaps for juggling with 3, 4, or 5 balls using throws of height at most 7.

`www.juggling.org/help/siteswap/` is recommended reading if you are not familiar with siteswap notation.

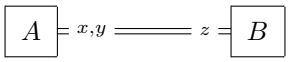
How to use

Decide how many balls you want to juggle and the highest throws you want to use, and find the corresponding diagram.

The **arrows** are important. Each arrow is labeled by a digit which represents a **throw**, or by a string of digits which represents a **sequence of throws**.

The numbers in the **boxes** are less important. They represent so-called **juggling states**, and the numbering is explained in the technical notes below. The state with the lowest number in each diagram is called the **ground state** and the others are called **excited states**.

Note: 

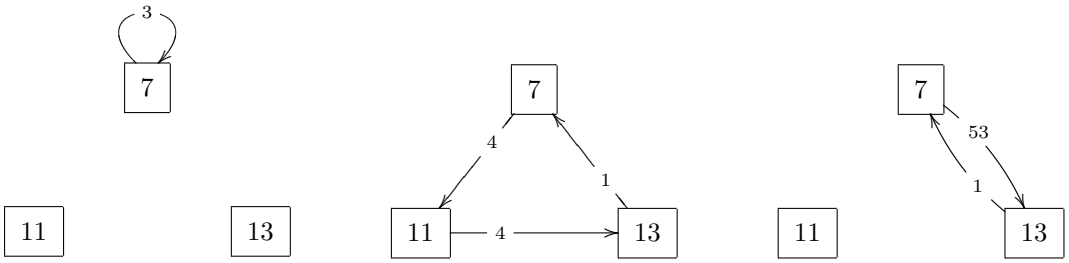
is drawn as 

in order to save space.

Any closed loop corresponds to a valid siteswap pattern, and vice versa. In other words: pick an initial state, then start walking around the diagram from there, following an arrow of your choice from one state to the next. For each step, remember the digit(s) on that arrow. When you return to the state where you started, the sequence of digits so obtained constitutes a valid siteswap. (And *every* valid siteswap with throws up to the chosen height can be found in this way.)

Examples

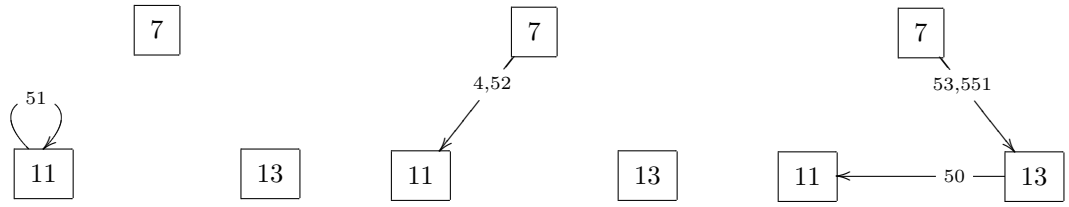
In these examples, I will use the diagram for **3 balls, up to height 5**. This is already enough for most well-known 3 ball tricks. The illustration below shows where the siteswaps 3, 441, and 531 can be found in the diagram.



Siteswap 3 (cascade) Siteswap 441 Siteswap 531

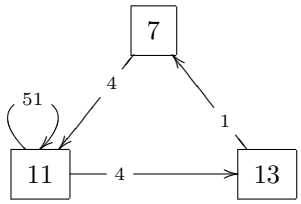
These three siteswaps all begin and end in the same state (namely the ground state 7). This means that they can be combined freely: 3 3 441 3 531 441, for example, is a valid siteswap.

The shower, on the other hand, starts and ends in an excited state, so one can't go immediately from a ground state pattern (like the cascade) to the shower; throwing 3 51 will cause two balls to land simultaneously in one hand. Some extra throws must be inserted as a transition between the two patterns, for example 3 4 51 or 3 52 51. As can be seen in the state diagram, there are four such transitions from the ground state 7 to the shower state 11 using throws no higher than 5:



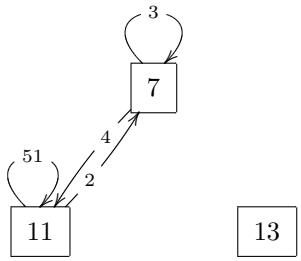
Siteswap 51 (shower) Transitions: 4 or 52... ...or 5350 or 55150

Inserting one cycle of the shower at the point where the siteswap 441 passes through state 11 produces the siteswap 4 51 4 1:



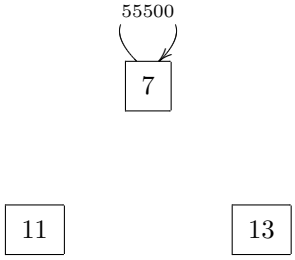
Siteswap 45141

The nice-looking number 12345 also happens to be a siteswap. When juggling this, it is better to think of it as 3 4 51 2 since that is a ground state pattern:

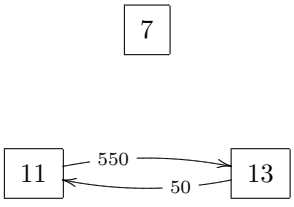


Siteswap 34512

Here are two ways of juggling 3 balls out of a 5 ball shower:



Three ball flash



The snake

Technical notes

These diagrams cover only standard siteswap juggling (asynchronous, no multiplex).

Juggling states are usually thought of as binary numbers with as many ones as the number of balls being juggled; a one at the kth position from the right means that a previously thrown ball will be landing k time units from now, a zero means no ball. In the diagrams, the states are labeled by the decimal and binary representations of that number.

The usual way of drawing state diagrams is to have each arrow denote only a single throw. However, the diagrams here are **reduced** in the sense that states having only one arrow coming in or only one arrow going out have been removed and replaced by extra (multi-throw) arrows. This leaves only the interesting states where there is a true "crossroads" of choices, and reduces the total number of states in the diagram from $\binom{d}{n}$ to $\binom{d-2}{n-1}$, where n is the number of balls and d is the maximum height:

	1	2	3	4	5	6	7	8	balls
height 2	1								
3	1	1							
4	1	2	**1**						
5	1	3	**3**	**1**					
6	1	4	**6**	**4**	**1**				
7	1	5	**10**	**10**	**5**	1			
8	1	6	15	20	15	6	1		
9	1	7	21	35	35	21	7	1	

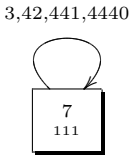
Number of states required in a reduced state diagram.

I discovered this simplification on my own, but later I found a paper on logic and juggling by Steven de Rooij[1] from 2001, where the possibility of making this kind of reduction was pointed out in a footnote. I also suspect that graph theorists have thought of similar things in non-juggling contexts long before that. So I don't claim to be the first one with this idea, although I might be the first one to really exploit it for juggling purposes.

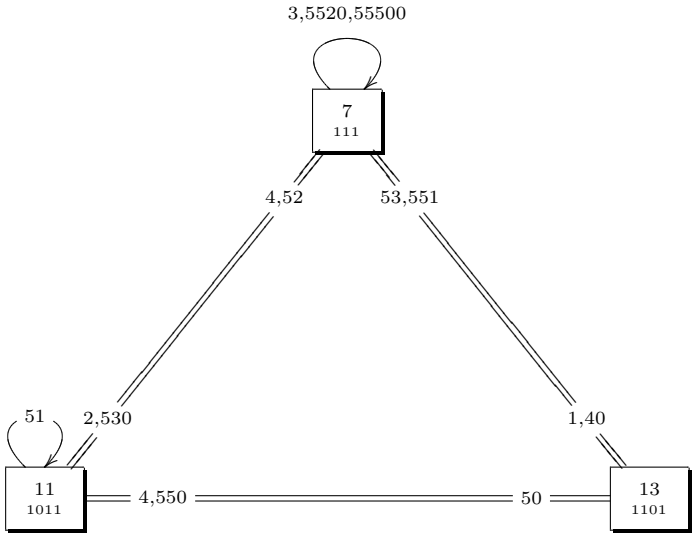
This document was produced using the LaTeX typesetting program with the XY-pic package. I wrote a Python program which generated XY-pic code for all the arrows and labels in the diagrams. The layout was then manually fine-tuned to avoid collisions. The first version saw the light of day on January 14, 2003.

[1] See `homepages.cwi.nl/~rooij/`.

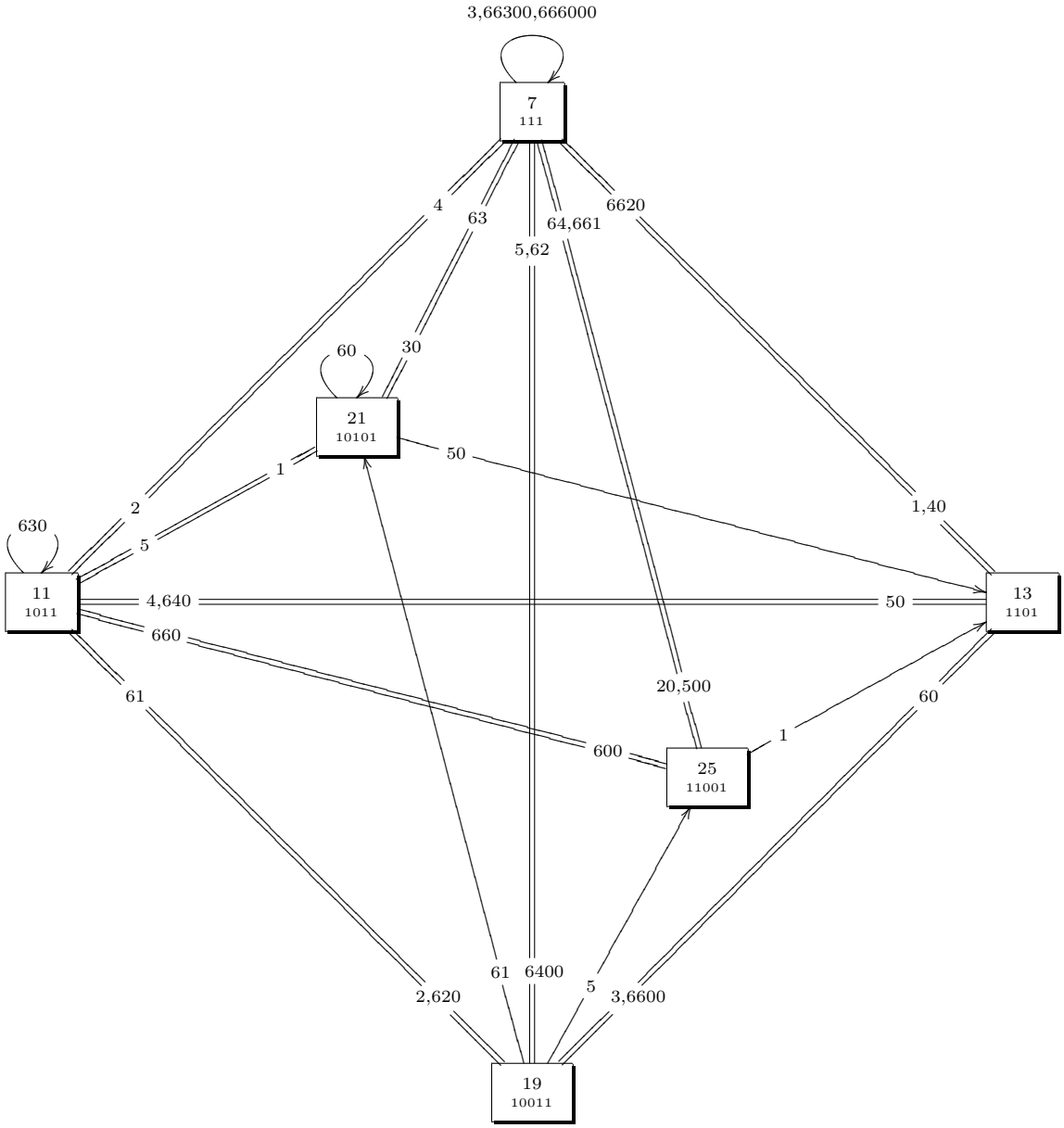
3 balls, up to height 4



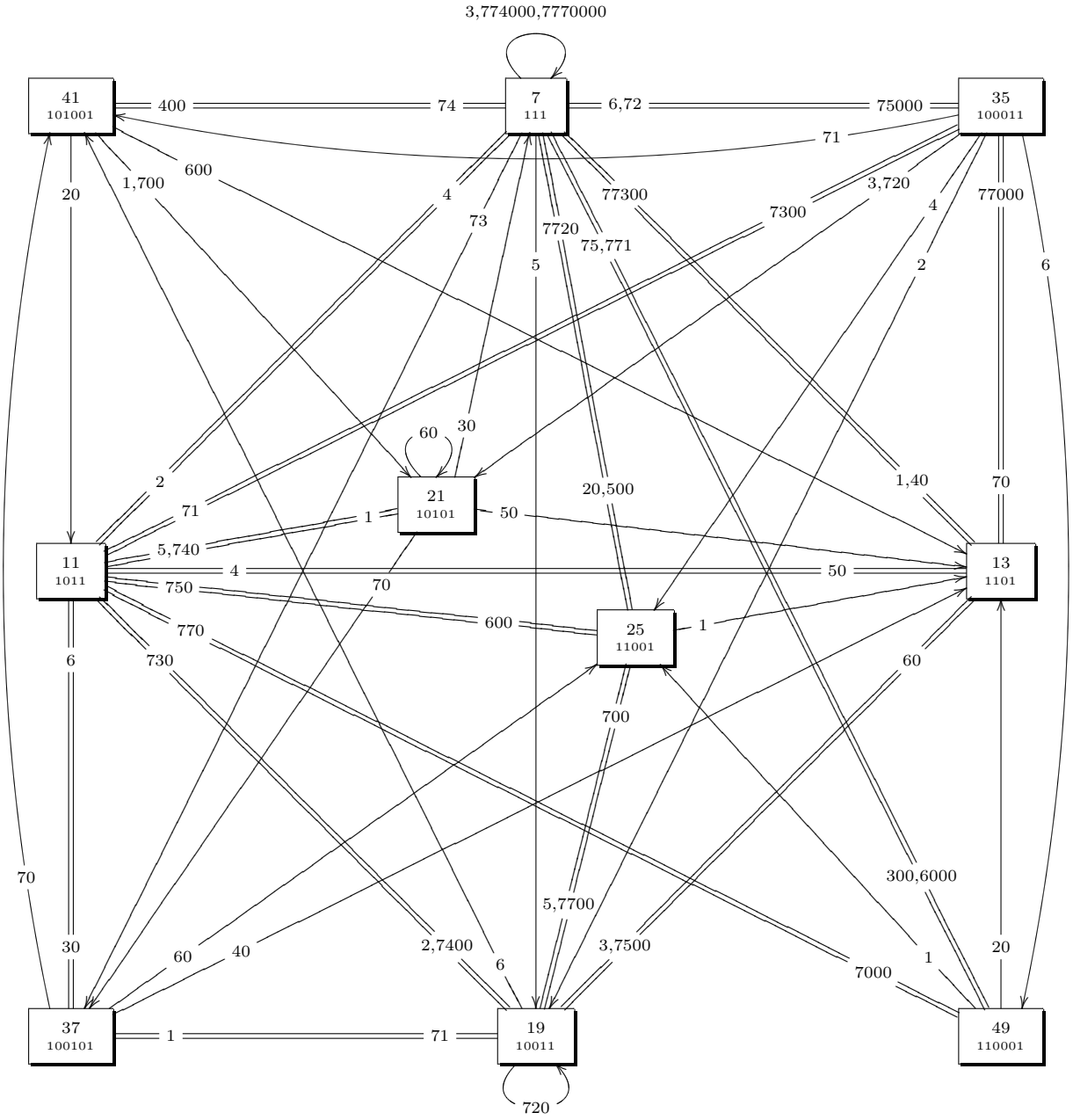
3 balls, up to height 5



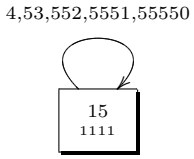
3 balls, up to height 6



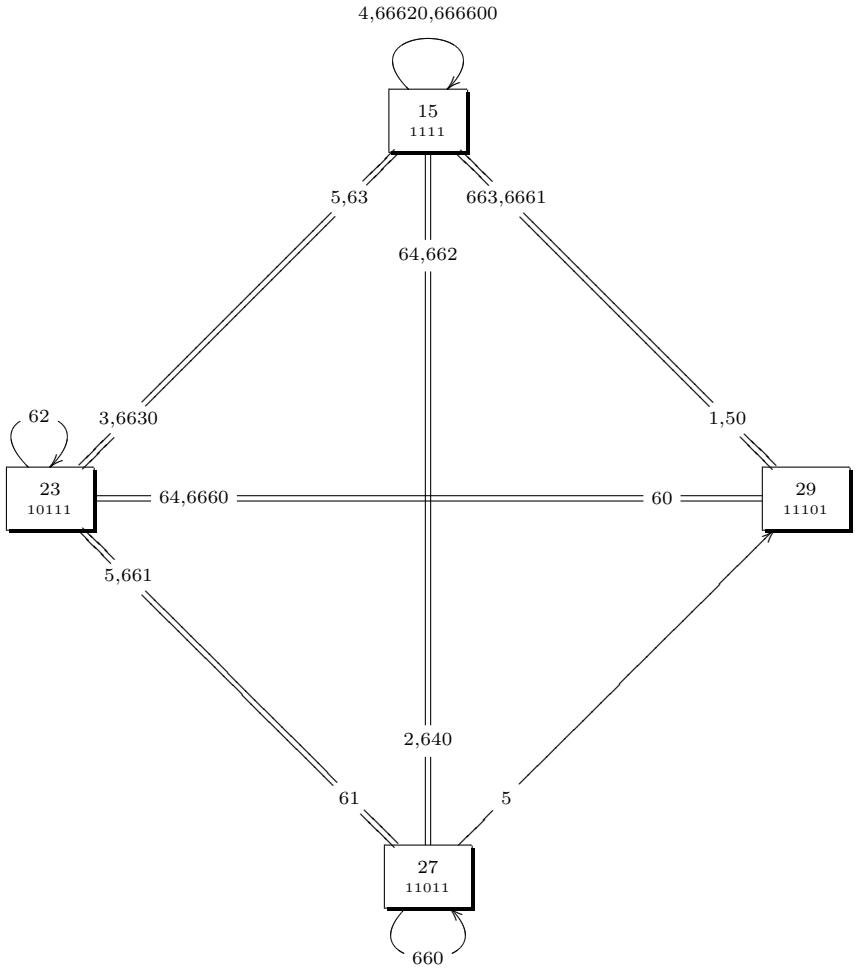
3 balls, up to height 7



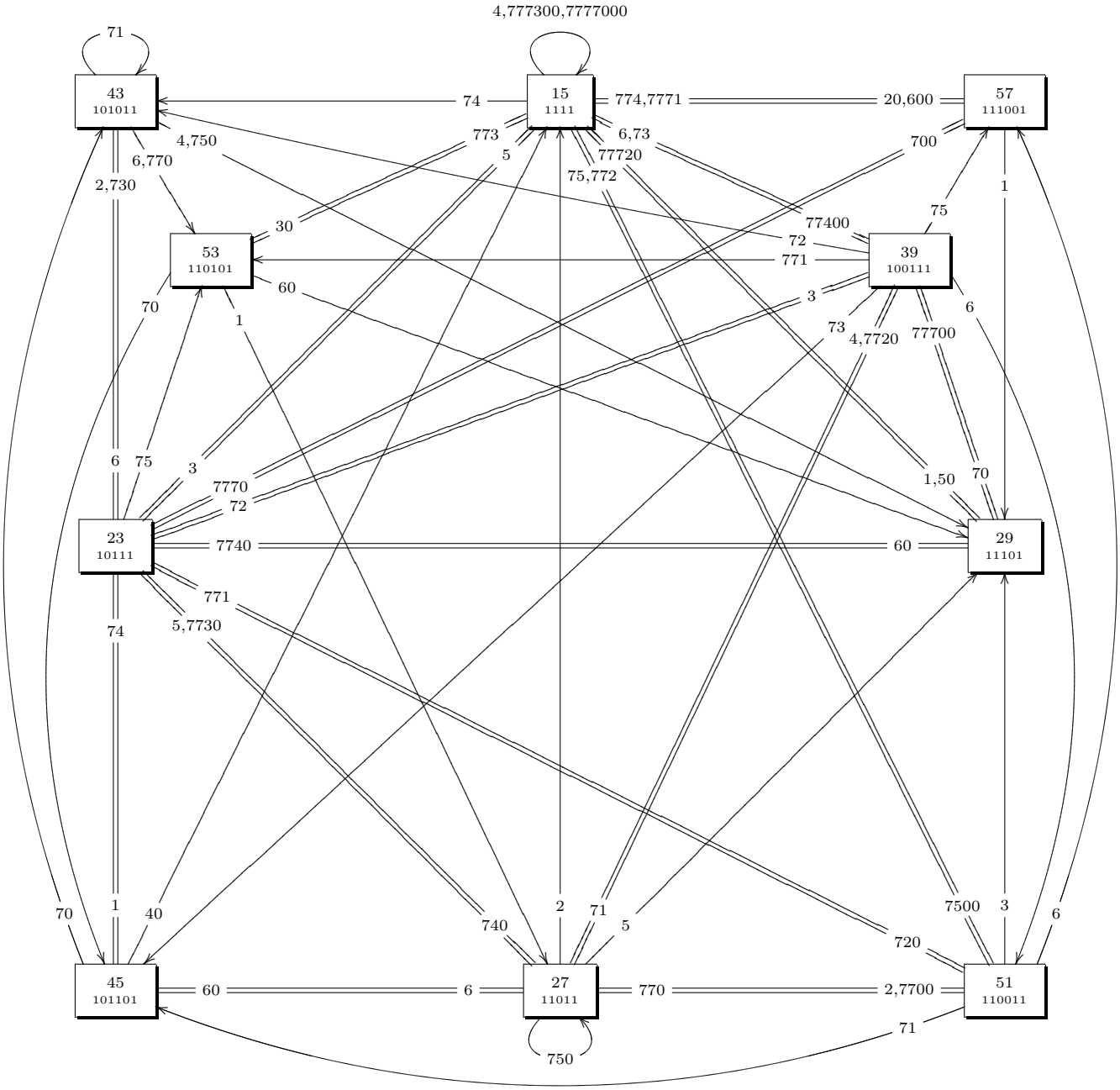
4 balls, up to height 5



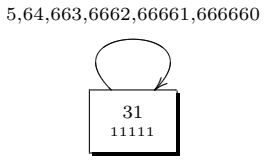
4 balls, up to height 6



4 balls, up to height 7



5 balls, up to height 6



5 balls, up to height 7